XXIII. *On the Determination of the Constants of the Cup Anemometer by Experiments with a Whirling Machine.*

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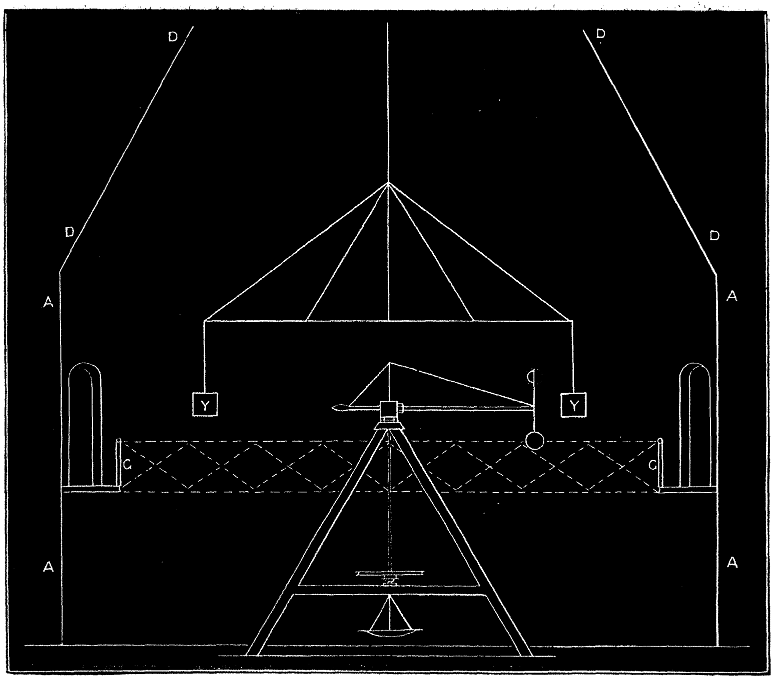
[PLATES 66-70.]

IN a communication to the Royal Irish Academy (Proceedings of the Royal Irish Academy, December, 1875), in which I examined the experiments made by M. DOHRANDT to determine the constants of the cup anemometer by means of ROBINS' whirling machine, I expressed a hope that I might have an opportunity of making similar experiments free from some influence which seemed to me objectionable. This opportunity was soon afforded me by a liberal grant from the Donation Fund of the Royal Society. I lost no time in having the necessary apparatus constructed by Mr. HOWARD GRUBB, of Dublin, and I now propose to give the results which I have obtained with it.

(1.) I was at first embarrassed by the difficulty of finding a suitable locality for the machine. In the St. Petersburg experiments the anemometers passed in their rotation at unequal distances from the walls of the building, and were too near its roof, which seemed likely to introduce extra disturbances into the air vortex which is formed by the whirl of the machine, and which, as will be seen, is a notable element of uncertainty in these investigations. But this was removed by Mr. H. GRUBB placing at my disposal the central dome of his works at Rathmines, then recently constructed for mounting the great equatorial which he is making for the Vienna Observatory. Its basis is a regular dodecagon, 42 feet least diameter, and 42 feet from its floor to the summit of the dome. Round it, at 10 feet from the ground, runs a gallery 2 feet wide, with a light iron railing, from which several doors give access to workshops in which polishing and other light work is executed. On the ground, open arches communicate with shops, in which large lathes and other heavy tools are established. This seemed all that I could wish, but, unfortunately, I could have the use of it only till the end of June, when it was wanted for the erection of the equatorial. There were, however, two inconveniences to be feared. One was that the open arches might admit irregular currents of air; the other was that when heavy turning or planing was going on, tremors were produced in the ground which might be expected to make the friction of the apparatus vary. I supposed, however, that this disturbance would equally affect the motion of the anemometers and the measures of their frictions; but this influence proved to be of far greater importance than I had anticipated. On the other hand, the vicinity of skilled workmen, ready to make any necessary repairs or any alteration suggested by my experiments, was of great importance, and not less so the help which Mr. GRUBB and his assistant, Mr. VEREKER, gave me during the whole of this work.

(2.) The vertical axle of the machine was placed in the centre of the polygon, supported by a wooden pyramidal framing. It is an iron tube $3\frac{1}{4}$ inches diameter and 10·4 feet long. It turns in a collar above and a perforated step below, supported by a transverse piece of the frame. On its top is fixed an iron box, 10 inches square and 12 inches high, open at two opposite sides. The third side supports, on a bar 40 inches long, a counterpoise disc of lead, 40 lb. weight, set edgewise. The fourth carries a horizontal arm of sheet steel rivetted together, of the section shown in Plate 67, fig. 1; at the box $5\frac{1}{4}$ inches by 3 inches; at its outer extremity 2 inches by $\frac{6}{8}$ths of an inch. In this turns the axle of the anemometer between three friction wheels at each end. These were at first of bronze, 1 inch diameter, but on May 22 the outer three were changed for others of hard steel $1\frac{1}{16}$ inch. On the top of the box is an upright, bearing a steel wire attached on one side to the counterpoise, on the other to the arm to prevent its flexure. The box also contains a brake apparatus (described at paragraph 21) and provision against centrifugal friction. The vertical axis carries near its bottom a driving pulley 24 inches in diameter, and both it and the horizontal axis are connected by circuit interruptors with a chronograph.



The centre of the horizontal axis is 14 feet from the floor, and the plane of the anemometer's arms is 9·0771 feet from the centre of rotation. Hence it follows that the cups of the instruments are 12 feet from the walls, 10 feet from the railing of the gallery, and, in the lowest case, 12 feet from the floor. The annexed sketch will show the disposition of the apparatus (except the driving machinery). A, A is the inner wall of the building; D, D a portion of the dome; G, G the gallery; Y, Y the vanes of the apparatus for measuring the air vortex subsequently described in paragraph 13.

(3.) The plan of having the plane of the anemometer's rotation vertical instead of horizontal, is liable to three objections, which will be noticed hereafter, but I was induced to adopt it on two accounts. Had the brake apparatus been at the outer end of the horizontal arm, the centrifugal friction would have been twice as great, and the action of the brake less certain; and secondly, the cylindric zone of disturbed air in which the cups move is little thicker than their diameter, while in the other case it would be nearly five feet across, and as the velocity of the air vortex decreases outwards, the uncertainty of its influence would be greatly augmented.

(4.) From the above we find that calling N the number of revolutions of the vertical axis in S seconds, the velocity of the anemometer's centre $V=\frac{N}{S}\times L^{-1}$ (1·58979) in miles per hour. The anemometers used had arms 2 feet and 1 foot long respectively. Calling A the turns in S, $v=\frac{A}{S}\times L^{-1}$ (0·93288) is the velocity of the centres of the cups in miles per hour; and half this for the shorter arms.

(5.) The revolutions of the vertical shaft and the anemometer are recorded on an electric chronograph a little modified from KRILLE'S construction; which was placed in one of the rooms opening into the gallery. It has three tracing points, of which the left records N by transverse marks on the helical line which it traces on the barrel of the chronograph as it is moved uniformly by a screw. This line is drawn on peculiar paper enamelled with some white composition, which is coated over with black. The point cutting through the black exposes a white line beautifully distinct on the dark ground.* The right hand point records A similarly. The middle point was connected with Mr. H. GRUBB'S regulator, and recorded S by interruptions of the helix, so that the three elements of the experiment were closely contiguous. The motion of the barrel was regulated by a governor clock, such as Mr. GRUBB applies to his equatorials, and its accuracy is very great, as is shown by the line of seconds; so great, indeed, that before long I dispensed with the seconds record, each revolution of the barrel being *exactly* a minute. There was attached, of course, a commutator, which brings each point separately into action.

(6.) The driving apparatus is carried by a vertical frame resting on the ground and attached at top to the gallery; it is an application of HUYGHEN'S maintaining power, and shown in fig. 2. A B is one of the top bars of this frame; it carries the driving pulley D and the free pulleys I, C, G, H, F, and K. The endless rope ($\frac{1}{2}$ inch diameter) passes from D round these. I and K are set obliquely to the plane of the

* This paper is supplied of excellent quality by Messrs. DE LA RUE.

others, so that the rope after leaving them and crossing, goes round the driving pulley on the vertical shaft. On the axle of D is fixed a large drum, E, which gives motion to it, being driven by a band from a PORTER's wheel. These pulleys are 15 inches diameter, and their spindles $\frac{5}{8}$ths of an inch. To G and H weights of 60 lb. are hung to maintain the requisite tension on the driving pulleys. If an additional weight, D, be hung to G, and the drum turned so as to raise it, it is obvious that when G descends the rope C I will be drawn up, F K drawn down, and H up; therefore the vertical shaft will be turned with a uniform speed by a force half D (neglecting friction) as long as G remains suspended. It works freely. When the anemometer was removed from the end of the horizontal arm, V=52 miles with D=20 lb. I made, however, a great mistake here, from motives of economy, in using manual labour as the motor of this apparatus, instead of adopting the suggestion of Mr. GRUBB of constructing an additional piece, by which the power of his steam engine could have been applied to drive it with any required speed. It would have been far less expensive than the labour proved to be; and I could have carried my experiments to a far higher velocity. When D exceeded 30 lb., two men were required at the wheel; and in one instance, where it was 105 lb., four men were required, and with difficulty could go beyond three minutes of the exertion.

(7.) Opposite the driving frame is a platform which gives easy access to the anemometer for measuring its friction, and cleaning or oiling the various bearings. It is a trapezium 10 feet long, the longer end of which is 8 feet long and is hinged to the platform of the gallery; the shorter end, which is 2 feet long, reaches towards the anemometer. When in use it is supported by a strut resting on the ground; at other times it hangs vertically, and its influence to disturb the air vortex was found to be insensible.

(8.) Five anemometers were experimented on. No. I. is of the Kew type; its cups 9 inches diameter, its arms 24 inches long; but instead of being of thin plates moving edgewise, it was necessary to make them of steel tube 0·5 inch external diameter, as stays could not be applied to oppose the centrifugal force, and even these were scarcely strong enough.* Its weight =110·87 oz. In No. II. the cups are 4 inches diameter, and the arms 24 inches; the weight =72·5 oz. No. III. has cups 9 inches and arms 12 inches; weight =68·25 oz. No. IV. has cups 4 inches and arms 12 inches; weight =40·5 oz. No. V. was tried as a matter of curiosity; its cups were semicylinders with mouths 9 inches square, the planes which closed their ends set parallel to the

* The equation (C) given when treating of centrifugal friction enables us to compute the outward throw due to this cause. One of the 9-inch cups weighs 12 oz., and its arm 15·7 oz. At the highest V which I obtained with them =36·29 miles this throw =124·8 oz. acting at their C G by a leverage =17·2 Now we found that 80 oz. acting on the arm by leverage =19·5 inches deflected the centres of the cups outwards 0·275 inch. This would make the real V in this case 0·091 mile greater than the estimated one. I did not, however, think it necessary to take this into account, as there are so many other and more important cases of uncertainty.

plane of their rotation; the arms (to the centres of the square mouths) 24 inches, and the weight $=129$ oz.

The other parts (beside the anemometers) which move with the horizontal shaft weigh 178·82 oz.

(9.) The method followed in experimenting was this. The machine was put in motion with as small a D as would make the anemometer revolve; when this had attained as uniform a motion as could be judged of by the eye, the electric circuit was completed by the commutator, and the action was continued generally for four minutes. During this time a person watched the chronograph to guard against its failing to record, which sometimes happened from oxidation of the contact maker, or from the points becoming blunt.* I made the observations for vortex motion, to be soon described, and Mr. VEREKER kept watch over the labourers.

When this observation was finished, a larger D was applied to give increased velocity, and so up to 76 lb., which I only passed in one instance. Even this was very severe on the men; the more so as the temperature of the place was often as high as 70°.

(10.) When this series was completed a weight was hung on the brake, and a new one taken. On account of the increased friction the anemometer would not move with the first D of the preceding series, and I adopted the plan of keeping the V's nearly equal. I laid down on a card the average length of the mark made by the point which records N in each experiment of the first series; and then set N alone to act; I altered D till its trace was nearly as long as the corresponding one on the card. Then the A point was made to record; and thus another series was completed. This was continued till the load on the brake was as large as could be used with safety to the levers in the case of No. 1; and with the others such as would admit of a sufficient value of v. The chronograph sheets, which are carefully preserved, were tabulated independently by two persons.

(11.) Before proceeding to discuss these observations, it is desirable to consider the conditions which determine the amount of an anemometer's motion. This obviously depends on the impelling force of the wind and the various resistances which oppose it. These resistances can only arise from the action of the anemometer itself on the air, and the friction of its parts; and therefore V, the velocity of the wind, is a function of v and F. The nature of this function cannot be determined *à priori* in the present state of hydrodynamics; but a general conception of its form is easily obtained. Suppose the wind makes an angle θ with the mouth of a cup (or the arm which carries it) which is revolving with such angular velocity that the centres of the cups move with the velocity v, I have shown in the paper already referred to that the velocity R with which the wind is incident on the cup $=\sqrt{V^2+v^2\mp 2Vv\sin\theta}$, the negative sign belonging to the first semicircle. The pressure of this to turn the cup$=R^2\times a_1$, a_1

* These points should have been diamonds, as in the chronograph at the Armagh Observatory, but they could not be immediately procured.

being the pressure of an unit of wind on the cup estimated in a direction normal to the arm. But as the arm carries on the other side of the anemometer's axis another equal cup equidistant and in a reversed position, the action of the wind on its convex surface will oppose the motion with the pressure $R'^2 a'_1$. The actual force therefore $=a_1(V^2+v^2-2Vv\sin\theta)-a'_1(V^2+v^2+2Vv\sin\theta)=(a_1-a'_1)(V^2+v^2)-2Vv\sin\theta(a_1+a'_1)$. This force is opposed first by the moment of friction at the centres of the cups, secondly by the resistances depending on v^2, of which the chief are the resistance to the arms and the reaction of the cups on the air which forms an air vortex in the plane of the anemometer. When these opposing forces balance each other through a revolution, the angular velocity of the instrument must be constant (except for small periodical fluctuations, the effect of which is much lessened by the moment of inertia of the arms and cups; still more so if four cups are employed). If we could find the mean values in a revolution of a_1; a'_1; $a_1\sin\theta$; $a'_1\sin\theta$, we could express this state of permanent motion by the equation $\alpha V^2-2\beta Vv-\gamma v^2-F=o$ (I.), in which F can be obtained by measurement. a_1 and a'_1 must be functions of the angles Ψ and Ψ' which the resultants R and R^1 make with the arm that carries the two cups, and which are given by the equation $\sin\Psi=\frac{V\sin\theta\mp v}{R \text{ or } R'}$.

(12.) But even in the case where the cups are at rest and $v=o$, we do not see our way to a determination of these functions. In the case of the concave surface, one would naturally suppose that when $\theta=180°$, the wind being parallel to the mouth of the cup can exert no pressure on it; but so far is this from being the case that a single cup will only be in equilibrio 30° beyond this position, notwithstanding the pressure on the convex surface. I cannot say how it behaves at $\theta=o$, for then the equilibrium is unstable and the cup gets into rotation. Yet more; the centre of the wind's pressure on the concave varies with θ; before 90° it is within the centre, after it outside; and its place depends on the deflection of the air stream-lines, the law of which is unknown. Equally uncertain is a'_1; but it is evidently a different function. It acts through the entire circumference; the surface which the convex exposes to the wind varies according to a different law, and the deflection of the stream-lines on it is of an entirely different character. I thought it possible that eddies might introduce terms depending on the first powers of V and v, but it will hereafter be shown that this is not sensibly the case; though the expression for Ψ implies that v should lessen α by diminishing the arc of θ through which R is effective. But as a_1 must be small at the beginning and end of the semicircle, it is possible that this influence is not important. These considerations, I think, justify me in believing the equation (I.) to be a close approximation to the general conditions of anemometer motion.

If we had a series of observations in which V′, v, and F were accurately known, we could determine by minimum squares the three coefficients α, β, and γ. If on applying these to the successive values of v and F, we reproduce the values of V′, the correctness of (I.) is established; if not, the march of the errors may enable us to trace whether they be casual or depend on extra powers of v; and I entertained such a hope when I entered on the experiments, but it has been but very imperfectly realised.

(13.) In the first place, we have instead of real wind the transport of the anemometer with the velocity V through air which is not quiescent, but moving in the same direction with a velocity W. Therefore we must use instead of V, $V'=V-W$. Here are two elements of uncertainty. It is not certain that a body moving through a fluid even if this be quiescent, is equally resisted with one at rest sustaining the impulse of a current fairly uniform, much less so if the fluid be in a state of disturbance; and secondly, though V and v are given with sufficient accuracy by the chronograph* it is otherwise with W. We cannot measure it in the actual track of the anemometer, and must reduce our measures to that track's centre on some hypothesis; while we may be sure that it varies in every part of the circumference described by the cups. But even in the line of its measurement it will be found very irregular and disturbed by powerful eddies; and besides these vorticose motions in the direction of V, there is another Z of very irregular character in a direction normal to this, so that the air moves in spirals not in circles, and instead of $V-W$, we should use $\sqrt{(V-W)^2+Z^2}$, but I see no possible mode of estimating the effect of Z correctly, on account of its intermittent character. WOLTMAN's fly, which was used by M. DOHRANDT to measure W, seemed unknown to our opticians; but at a latter period I was informed by a scientific friend that it was called here an air meter, and he lent me one, which I found useful. But I could scarcely have used such a one for habitual measurement without chronographic registry, and I wished for something that would show the changes of the vortex more evidently. Fig. 3 (Plate 70) shows the method I adopted. A slip of deal, $\frac{1}{4}$ inch square and 23 feet long, was suspended by a fine thread from the summit of the dome; to prevent bending, it was braced by other threads fastened 4 feet above it; to it were suspended two of those thin caoutchouc balloons which are sold for playthings for children. They were about 8 inches diameter and a little higher, and hung 4 feet below the rod, their centres on a level with that of the anemometer, and 14 inches outside it. Threads connecting them with the ends of the rod prevented their being drawn in among the cups. From this it will be seen

* A second of time measures on the chronograph 0·208. Now the highest value which I obtained for N was one revolution in a second, for v 1·67; so that even in this last case $\frac{1}{10}$th of a unit of $A=0{\cdot}012$ inch, a quantity quite visible. It must, however, be remarked that my V belongs only to the cup whose arm is perpendicular to the horizon, whose $\theta=90°$. For any other θ it $=V(1+\frac{1}{2}\frac{r^2}{R^2}\times\cos^2\theta)$. For Nos. I. and II. this is $1+0{\cdot}243\times\cos^2\theta$. At its maximum the addition is $\frac{1}{41}$ of the whole; in its mean value through the semicircle half this. But this increase of V is counteracted by its obliquity to the plane of the anemometer motion, which lessens its power to turn the cups. The tangent of this $=\frac{r}{R}\cos\theta$; it$=12°\cdot26'$ at maximum. If we knew the forms of a and a' we could compute the effect of this obliquity; but as it is, we can only say that both these disturbances are greatest when a and a' are least, so that probably their influence may be neglected.

that the free part of the thread is 23 feet long, and its torsion force is insensible in these experiments; after being twisted during several series when left to itself it resumed its original position so slowly that ten minutes were required for about a third of a revolution. The lightness of the balloons and their large surface make them obedient to very slight currents, and extremely sensitive to any irregularity in its motion. The process of observing W is this: When the experiment has lasted for about a minute, the time b of two revolutions of one of the balloons was noted by a stop-watch (by ROBERT, of Paris) whose beats are $0{\cdot}2^s$. From the radius of the circle described by them, we find that W in miles per hour$=L^{-1}\frac{(1{\cdot}94335)}{b}\times$reduction to cross. This reduction is made on the assumption that the velocity in any part of the vortex is inversely as the square of the distance from the centre. This agrees as well with experiment as under the circumstances could be expected. Thus on one occasion, at the normal distance $b=83^s$, when the distance was increased to 30 inches it became $104{\cdot}6^s$. The inverse square gives $106{\cdot}2^s$. Hence the factor for reduction $=L^{-1}(0{\cdot}10586)$. These balloons were very perishable; drops of oil were projected occasionally from the bearings at high speeds, which perforated them like shot, and by the commencement of June we had exhausted the stock in the neighbouring shops, and most of those in Dublin. They were then replaced (June 7th) by vanes of tissue paper 1 foot square, stiffened at their edges by rods of deal 0·125 inch thick, and suspended from the rod by slips of deal 0·5 inch broad, 0·062 inch thick, and 4 feet long. These moved edgewise and were kept steady by lateral slight trusses of the same material and scantling. As the vortex reaches far above the rod these suspensions will not sensibly retard the vanes. Their centres are 20·25 inches from the cross, but they give b 0·003 smaller than the balloons. Their reduction to the cross $=L^{-1}(0{\cdot}14810)$.

(14.) This apparatus shows that the vortex current is very irregular: the balloons sometimes dart rapidly forwards, then move more slowly, and this not only when the anemometer is passing them; sometimes they are violently projected upwards and outwards, showing that in addition to the general translation of the air, it is affected by both horizontal and vertical eddies of considerable power and possibly magnitude. The vanes can only show irregularities in the circular motion, but fully confirm its lawlessness. Under such circumstances we can have no exact knowledge of the W which exists *at the cups;* the balloons give its mean value through the circumference, but it is probably greatest immediately after the cups have passed, and decreases by the eddies and the friction of the walls till they again meet it. Any fixed meter would yield the same result, and the only proper plan would be one capable of giving the value of W at the cups for every instant. A pressure gauge carried with the cross and connected with a chronograph promises best; but in air so agitated the relation between the velocity and pressure would be very doubtful.

(15.) The horizontal arm has little effect in producing the vortex; in the case already mentioned where it was whirled for several minutes without the anemometer, scarcely any motion of the balloons was observed though V=52. Nor does the anemometer's own rotation seem to have more effect, for $\frac{V}{W}$ does not change with a given value of V though v be reduced by brake friction to half its original amount; it decreases, though slowly, with V. Out of many comparisons, those given in the following table may suffice:—

TABLE I.

D=75 Brake.	V	$\frac{V}{W}$	D	Mean V.	Mean W.	$\frac{V}{W}$	No. of Observations.
0	29·81	12·79	75	29·53	2·213	13·34	8
6	29·49	12·66	65	27·15	2·128	12·75	8
12	29·81	12·20	57	25·40	2·016	12·60	8
18	29·81	12·71	46	21·66	1·825	11·87	7
24	29·97	13·18	39	19·71	1·689	11·67	7
30	29·17	12·64	28	15·19	1·370	11·08	7
36	29·25	12·96	23	12·57	1·296	9·67	5
			20	10·40	1·039	10·01	4
			18	8·705	0·923	9·43	4

The three first columns belong to those single observations of the second set of No. I., in which the driving weight was 75, and allowing for the irregularities which I have described, may be considered to show that $\frac{V}{W}$ is constant. The remaining columns relate to all in the first set of No. I., where the driving power is the same without regard to brake friction; here the decrease of the ratio is obvious, though it might not have been evident from single observations. For instance, the first of the eight ratios belonging to D=75 is 11·89, the fifth is 14·30.

(16.) I obtained some additional information about the vortex by the air meter already mentioned. It is a small windmill of eight vanes (five would have been better), 2·8 inches diameter, connected by a stop and wheelwork with a series of indices registering its revolutions to a million. It is very sensitive, though less so than the balloons or vanes. A revolution is stated to be a foot; but there was in its case a table of corrections which seemed to me so strange that I did not use them. Attaching it to the horizontal arm without the anemometer, and whirling for 8^m with V=17, I found a division =0·867 foot. On placing the meter on the platform with the necessary precautions, with its axis of rotation in the plane of the arm and in the direction of W at 14 inches from the cross, I got in 2^m 212 divisions with D=35. This on the above scale is 1·53 foot per second, the balloons at the same time giving 1·57 feet. It is, however, needless to turn the divisions into feet, as they serve for comparison. When the meter was placed to measure the radial current it gave 70—a third of the other—but with this unexpected peculiarity: that during the two minutes it sometimes moved very rapidly and sometimes almost stopped. In another trial with the same D at 28 inches distance the tangential velocity=162, the radial 60; with D=22, distance=20 inches, tangential 122, radial=82 with the same irregularity. It follows from this that the vortex motion is not, as I had supposed, in a regular spiral, but in a track of very complicated character, which it is not practicable to define. At the same time I tried to verify with this instrument the law of inverse squares, which cannot easily be done with the balloons, as it was very troublesome to shift them. A scale was fixed on the platform, so that the meter could be exactly placed at any distance from the cross. Four distances were taken for each value of D, and the machine was whirled for 5^m before beginning to measure, and kept in motion till the four were finished. The irregularities are considerable, partly owing to the small size of the vanes, for even in real wind the current is far from uniform, narrow veins of great intensity being contiguous to others comparatively feeble. The results are given below with their reduction to the cross by the factor $\frac{(108+d)^2}{(108)^2}$.

TABLE II.

	D=25, V=15·55.			D=30, V=17·82.			D=35, V=19·12.		
Dist.	Meter.	Reduced.	Do. −M.	Meter.	Reduced.	Do −M.	Meter.	Reduced.	Do. −M.
12·25	233·0	238·9	+1·1	274·0	339·7	−34·6	291·0	360·7	−1·4
	−38·5			−57·5			−40·5		
20·25	194·5	274·3	−13·5	216·5	307·3	+2·2	250·5	353·2	−9·9
	−15·0			−24·5			−1·5		
28·25	179·5	285·7	−2·1	192·0	305·6	+0·5	243·0	386·8	+24·7
	−10·0			−42·5			−48·0		
36·25	169·5	302·4	+14·6	149·5	267·9	−37·2	195·0	347·9	−14·2
		287·8	0·00		305·1	−17·3		362·1	−0·2

The irregularities of the differences show that the errors of the formula are merely casual.

(17.) Another question occurred to me: Does W go on increasing for any considerable length of time? To ascertain this a set were recorded during ten successive minutes as soon as the anemometer got into full motion; the b's are separated by the time of one revolution; the circumstances were very favourable, and the results are a good specimen.

TABLE III.

No.	N.	A.	b.	V.	v.	W.
I.	28	45	78·8	18·15	6·43	1·421
II.	28	46		18·15	6·57	
III.	28·17	45·5	71·4	18·15	6·50	1·566
IV.	28·17	45·25	66·4	18·26	6·46	1·689
V.	28	45		18·15	6·43	
VI.	28	45	66·4	18·15	6·43	1·689
VII.	28	44·5		18·15	6·36	
VIII.	28	45	68·2	18·15	6·43	1·643
IX.	27·5	44·8	66·4	17·82	6·40	1·689
X.	27·5	44		17·82	6·28	

W becomes sensibly constant after four minutes, and this time was allowed in subsequent experiments.

(18.) On the whole, the estimation of W is a weak, perhaps the weakest, part of this investigation; and what I have observed respecting this air vortex makes me doubt whether the conclusions deduced from such experiments can be fully applicable to real wind. Still, it may be hoped that they will give useful information.

(19.) Friction is an important agent in anemometric motion, and one in which I found more difficulty in the measuring than I had anticipated. It was fourfold: (1°) that due to the instrument's weight; (2°) that caused by the application of the brake; (3°) that caused by the lateral pressure of the axle on the upper bearing, caused partly by the action of the wind, partly by the force which, as in the gyroscope, is required to change the plane of the anemometer's motion; and (4°) that due to centrifugal force. There are three degrees of this resistance; the highest is that which occurs when a machine is started from rest; the intermediate is that which acts during continuous motion, which is the one to be considered here, and the least which prevails when the parts that rub are put into a state of vibration by tapping or jarring, which may be sometimes only half the first. Friction occasionally varies from the effect of temperature or chemical change on the oil used to lubricate the bearings, or else of dust finding its way into them (as sometimes happened).

(20.) Nos. 1 and 2 were measured thus. A cast-iron disc, weighing 103·17 oz., was fixed on the shaft in place of the anemometer; on its circumference a groove was cut whose bottom was 1·0 foot diameter, in which a fine silk thread was wound; to this, weights were appended till when the disc was barely moved they continued the motion. This weight, divided by four, is considered to represent the moment of friction at the centre of the cups after a correction for the increase of friction due to its own pressure.

1. The normal friction f is larger than in ordinary instruments on account of the long shaft, its horizontal position, and the weight of the brake apparatus :* with the disc in place the moving parts weigh 282 oz. The measurements at Rathmines gave values from 100 to 120 grains; but I adopt one which I obtained at Armagh by a better process: that used by ATWOOD in determining the constants of his celebrated machine. If a weight W be hung to the disc, $W-f$ accelerates a mass $W+M$; M being the moment of inertia of the moving parts reduced to the circumference of the disc, and we have by the law of uniformly accelerated motion $g\times\left(\frac{W-f}{W+M}\right)=\frac{2S}{T^2}$, S being the space through which W descends in the time T. We get S most easily by taking a given number of the disc's revolutions. M was got by suspending the disc and its appendages in the sector tower of the Observatory with a bifilar suspension whose lines were 62·5 inches long and 3·01 inches apart. A graduation fixed on the disc enabled me to note the arcs of vibration; 300 complete vibrations were taken for each trial, and by the usual formula x^2 the square of the distance of the centre of gyration was found. $\frac{\text{This}\times W}{36''}$ gave by ten sets M=25,773·6 grains. Hence we get f far more consistently than in the former method.† I obtained 72 observations with W from $\frac{5}{8}$th of an oz. to $1\frac{1}{4}$ oz., and divided them into three groups to see if f varied with the velocity. The first 24 gave $f=114·18$; $v=1·33'$. The second gave $f=110·41$; $v=2·13'$. The third $f=108·20$; $v=2·87$. There is a decrease, but I think it is mere error of observation, for the effect of an error in noting the time acts inversely as T^3;‡ so I take the simple mean=110·93. To obtain from this the f for each of the anemometers, 2 lb. were hung on the axle, and ten observations gave

* In its present position with its axes vertical the normal friction is only 22 grains.

† As the rotation of the disc is liable to be disturbed at its commencement by any casual irregularity of the friction, it is better to reject this part and take two spaces immediately consecutive. In this case it is easily shown that $g\frac{(W-f)}{W+M}=\frac{2S't-2St'}{tt'\,(t+t')}$, or (if $S'=S$)$=\frac{2S\,(t-t')}{tt'\,(t+t')}$. Both S and t are given by the chronograph, of which, however, I could not then avail myself. This method has over that which I used at Rathmines the advantage of giving the friction during actual motion over a considerable space. If when the descent of W has given a rapid angular motion to the disc it be detached from it, the disc will continue to revolve till stopped by the friction. This is equivalent to supposing W=0 in the equation; and if f be so small that it makes twenty or thirty revolutions, it is ascertained with great precision. It is much to be regretted that this process did not occur to me at Rathmines, for it would probably have very much improved my results. Had I then possessed my present knowledge, I would have made an arrangement to give *the mean friction during each experiment*, which could have been effected by a modification of the means (hereafter described) used to measure a alpha.

‡ Some observations made with the anemometer's axis vertical show that f does not change with v, at least within the limits of 29·6 and 7·1 miles.

increase of $f = 9·7$ grains for the added pressure $= 0·294$ grains for every ounce, or 4·71 for every pound. Hence for the different anemometers—

No. I.	$f = 113·19$.
No. II.	„ 101·27.
No. III.	„ 201·32.
No. IV.	„ 184·98.
No. V.	„ 118·53.

(21.) 2. *Brake friction.*—This is produced by means shown in Plate 70, fig. 4. A is the box mentioned (2), to the bottom of which is screwed the strong brass frame *b*. In this turns the lever D, with its arbor E. The longer arm of this lever is connected with the piece F, slotted to give room for the shaft and one of its three friction wheels, as shown in the figure, and attached below to the wire W, which passes down through the vertical axis and its perforated step, and carries a scale-dish below. The shorter arm of D is connected by the link G to the right angled lever HH′, whose arbor is at L. If now a weight be placed in the dish it draws down F, and thereby D, and G raises H, makes H′ press the piece M, which turns with the arbor N, and presses the rubber R which it carries against the friction-disc C, six inches in diameter. A duplicate of this mechanism is placed opposite (but not shown, to avoid confusion), and being also actuated by the wire W, presses a similar rubber against the corresponding part of C. K is one of the counterpoises to balance the scale-dish, which, however, was much too heavy, as I had no idea what weight might be required, and it produced unnecessary friction. The rubbers are of stout cloth, 1·25 by 0·8 inch, and show no signs of wear. The leverage of the brake is $\frac{3·1}{1} \times \frac{6·5}{1·7} = 8·03$. By this elegant arrangement (for which I am indebted to Mr. GRUBB, F.R.S.) the friction, while it can be varied at pleasure, causes neither vertical or lateral pressure on the supports of the shaft. When, therefore, the brake is made to act, it merely adds to f another friction, which can be similarly measured. The measures, however, of this friction were much more discordant than those of f: two differing in one instance 234 grains. This irregularity is probably caused by the tremors of the ground (already referred to) acting on the brake-weight; when thrown upwards it would relax its pressure momentarily, and permit the measuring disc to revolve with a less pull than what really represented the friction.

When I measured this by ATWOOD's process at Armagh, the results were very regular, but far larger than those obtained at Rathmines, except on one occasion when Mr. H. GRUBB took a set very early in the morning before there was work going on. This agreed well with *them.* As, however, all the anemometer experiments were affected by this vibration, I deemed it best to use the Rathmines frictions.

But I found an unexpected fact: that the coefficient of *this* friction is not constant, but decreases with the pressure; this is probably owing to the elastic nature of the rubbers, which becoming condensed by the pressure tend to act like a hard body. On laying down the curve of this coefficient, it looked so like an equilateral hyperbola with coordinates parallel to its asymptotes, that I tried the Armagh values of it by the equation $\frac{F}{B}=\frac{y}{B+u}+x$, B being the load on the brake in ounces -200 grains, the force required to bring the rubbers into contact; each value of $\frac{F}{B}$ gives an equation of condition, combining which we get values of the constants. Substituting these in each, we get values of $\frac{F}{B}$ whose errors enable us to approximate still more by the equation $\frac{dF}{B}=dx+\frac{dy}{B+u}-\frac{du}{(B+u)^2}$. Thus we obtain $y=315{\cdot}52$; $x=77{\cdot}814$; $u=6{\cdot}662$. These give when $B=0$, $\frac{F}{B}=125{\cdot}28$; when $B=\infty$, $77{\cdot}81$; and afford values of F differing in most cases from the observed ones far less than the probable errors of the latter. I have therefore used these values in reducing the observations. The following table gives these: their differences from the observed ones, the number of observations, and $\frac{F}{B}$ for each value of B.

TABLE IV.

No.	On Brake.	Computed Friction.	$o-c$.	No. of Observations.	$\frac{\text{Friction}}{\text{B}}$
	oz.				
1	3	284·75	+ 2·3	7	113·03
2	6	574·3	+13·1	6	105·85
3	9	841·7	−18·2	4	96·42
4	12	1098·1	−14·0	7	93·51
5	15	1347·6	+11·2	5	93·45
6	18	1593·5	−15·9	5	89·95
7	24	2026·9	+13·1	4	86·66
8	30	2556·0	− 8·5	5	86·24
9	36	3031·1	+ 7·5	5	85·50

3. A lateral friction is produced by the wind pressing the anemometer's axis against its outer bearing. Omitting the consideration of the arms, this pressure will be the sum of the mean pressures on the cups during a revolution, and the same reasoning as in the case of I. shows that it $=\epsilon V'^2-2\kappa V'v+\epsilon v^2$. If, therefore, the constants of I. be determined by the observations, the effect of this friction will merely be to diminish α and β and to increase γ; ϵ and κ are larger than the other, and κ much less than ϵ, so that probably $2\kappa V'v-\epsilon v^2$ is small in comparison of the first term, and the pressure P is simply $=\epsilon V'^2$. This is not merely confirmed by these experiments, but a good measure of it is obtained. In these the force which turns the vertical axle is $\frac{1}{2}D$—the frictional resistance of the driving apparatus: this when the motion has become uniform, acting at the end of the horizontal arm=air's resistance=P at the point of

bearing. Now if we examine Plate 67, fig. 2, it is obvious that the spindle of the pulley G is pressed by the weight W+D; that of C by the same —friction of G, and so on to I. From this the tensions of the cords GC, CI, and of U, the cord which leaves I for the driving pulley can be determined. The same thing can be done for U′, FK, and HF. Now tension of U—tension of U′—friction of vertical axle is the force which drives the axle; and if this be worked out, supposing f equal in each pulley, and so small that powers above its square may be neglected, we have $U-U'=\frac{1}{2}D-(2W+D)\times f$. When D=12 the axle sometimes moves, but seldom; I think 11 is the limit, and as 2W=120 we have $f_1=\frac{5\cdot5}{131}$, and therefore $P=\left\{\frac{1}{2}D-(2W+)\times\frac{5\cdot5}{131}\right\}\frac{1\cdot025}{9\cdot078}$, and we can try if $\frac{P}{V'^2}$ is constant. This proves to be the case as is seen in the following table, which gives the results with Nos. I. and II.

TABLE V.

No. I.

D	P	ϵ'.	$P-\epsilon V'^2$.	No. of Observations.
22·3	0·583	0·004278	−0·017	3
28·5	0·903	0·004464	+0·015	4
33·8	1·174	0·004418	+0·004	5
44	1·700	0·004531	+0·048	6
55	2·267	0·004412	+0·006	7
64	2·730	0·004479	+0·047	7
74·3	3·260	0·004380	−0·018	7
105	4·843	0·004239	−0·186	1

No. II.

D	P	ϵ.	$P-\epsilon' V^2$.	No. of Observations.
25·75	0·760	0·001332	−0·051	4
28·8	0·917	0·001370	−0·002	5
35·8	1·277	0·001383	−0·006	6
40	1·507	0·001332	−0·062	6
48	1·906	0·001464	+0·014	7
56	2·318	0·001441	+0·078	1

The agreement is good; for the 9-inch cups $\epsilon=0\cdot004400$; for the 4-inch$=0\cdot001387$; and I think it likely that they would answer well even for real wind.

(22.) The observations for determining α (*vide* par. 27) show that the effect of v may be neglected, as in them $v=0$. I take three from No. I.

D	V′	P	$P-\epsilon V'^2$.
20	9·49	0·464	0·067
29	14·36	0·993	0·047
52	21·50	2·112	0·079

It will be observed that for the smaller cups ϵ is relatively larger than in the proportion of their areas. This is, in great measure, owing to the arms, 0·5 inch diameter and 24 inches long, bearing so much greater proportion to them. From the preceding it follows that $f''=\epsilon V'^2\times 4{\cdot}71$.

As to what I call gyroscopic friction, for want of a better name, the pressure which produces it $=(A-C)(\omega_{,}\ \omega_{,,,})$; A and C being the moments of inertia round the principal axes, and $\omega_{,}\ \omega_{,,,}$ the angular motions round them. C was determined, but it would be rather awkward to get A. As, however, the angular velocities are as V and v, the effect of this friction will be merely to lessen β.

(23.) 4. In the rotation of an anemometer at the end of a revolving arm, the centrifugal force produces an outward pressure which must be resisted by some stop, and produces there a friction f'''. This pressure is given by the formula (C) $P'''=\frac{M'\times G}{g\times R^2}\times V^2$ (not V'^2), where M′ is the weight of the parts which revolve round the axis of the anemometer, G the distance of the centre of gravity of M′ from the vertical axis, and R the length of the horizontal arm. This pressure was in some of these experiments considerable; in No. 133 it was 52 lb.; on another occasion it broke a steel arm of No. 111, $\frac{3}{8}$ths of an inch thick. As the resulting friction varies nearly as V'^2, it might, like f'', have been included in α; but as I hoped that the results of these experiments might be available for real wind (where f''' has no place), I thought best to measure it and add it to f. I determined it thus: A strong upright was fixed in the gallery, and secured by a strut to the platform; to this was fixed a pulley 1·73 inch diameter, as nearly as the eye could judge in a line with the seam of the horizontal arm, which was my nearest guide to the direction of the axis of the shaft. Over this passed a fine iron wire attached to the centre of the shaft, to which weights were suspended which pulled it in the direction of its length. The friction thus produced was measured by weights placed in the cups, and the normal f subtracted from this. The tensions were corrected for the friction of the pulley, which when this line crossed it at right angles was $\frac{1}{20}$th of the load. The only other mode of measuring this friction was the setting the arm vertical with the 12-inch disc below, placing weights on this and proceeding as in the case of f. This, however, would have required the total dismounting of the apparatus and constructing a proper stand, for which time could not be spared.

With $P'''=10$ lb., $F=180$ grains; with $P'''=21$ lb., $F=280$, each a mean of three trials. These give $\frac{F}{P'''}$ in the first instance$=7{\cdot}032$, in the second$=8{\cdot}361$. It seems from this that here the coefficient of friction *increases* with the pressure. I see no mechanical reason for this, but must accept the result, for the difference is clearly marked. The outward pressure was resisted by a single roller bearing on one side of a plate fixed to the shaft; this must have tended to make the latter press more on its bearings, and as it acted on the plate 1·6 inch from its centre, the friction was needlessly great. Therefore on May 18th it was replaced by two opposite ones, larger, carried by the shaft, and bearing on the back plate of the frame 0·5 inch from the centre of the shaft. This reduced the friction by 0·6 of its first amount. The measures were now repeated. With P=56 lb., F=277·5 grains; with P=70 lb., F=392, each also a mean of three. Here, however, f must not be subtracted; for with the two rollers the pull raises the shaft sensibly from the outer bearing. These give for the first $\frac{F}{P'''}$ 5·216, for the second 5·895. The simplest mode of representing these is the formula $F = P'''x + P'''^2 y$. These give for the first set (which were used till No. 79) $x = 5·8232$; $y = 0·1272$, and for the second $x = 2·5010$; $y = 0·0509$.

All the constants of the equation (C) except G have been already given; I give it here, and with it the factors for P‴ for each instrument.

No. I.	G=5·4605	$P''' = V^2 \times L^{-1}$ (8·56260).
No. II.	5·0232	L^{-1} (8·46338).
No. III.	5·0482	L^{-1} (8·53793).
No. IV.	5·0099	L^{-1} (8·40158).
No. V.	5·5965	L^{-1} (8·60018).

(24.) I suppose this variation of the friction is somehow connected with the rollers, but both COULOMB'S experiments and railway experience indicate that rolling friction is simply as the pressure.

(25.) On the whole, I consider that these friction measures, though perhaps not so uncertain as those of W, are yet sufficiently so to increase materially the difficulty of drawing accurate conclusions from these experiments. I expected that increasing m by additional friction would give a wider range to the coefficients of the equations of condition, and this is so. But experience shows that the advantage thus gained is neutralised by uncertainty when the value of v is small, for then the latter is greatly affected by the irregularities of friction to which I have already alluded. This probably arises from the momentum of the apparatus not being sufficient to overcome any casual increase of friction. But it is also possible that as the friction when a body is started from rest is greater than when it is in motion, the passage from one state to the other may be gradual through a certain small range of v.

(26.) Lastly, I shall describe the means which I employed to determine directly the coefficient α of equation (I.) and the results which they gave. The outer end of a strong clock-spring was attached to an arm of the anemometer set horizontal, the inner end to the horizontal arm of the machine: a circle divided to 100 parts was fixed on the shaft and read by an index on the arm. I hoped the tension of this spring would be so nearly as the angle of its torsion that mere reading of the circle would give it; but this was not quite the case, and I had to form a scale for it. The instrument was turned through a quadrant, and held while weights were hung on the outer edge of the cup till they balanced the spring. As the friction caused some uncertainty, the cup was a little raised and allowed to descend gently, and its point of rest read; it was then depressed and allowed to rise, the mean of the two readings being considered the true point of equilibrium. This was repeated for the second quadrant, and so on for three complete revolutions. The numbers so obtained were reduced to the centres of the cups, and corrected for the angular deviation from the horizontal position. The same spring was used for No. II. and No. III. with separate scales; but for No. I. it was necessary to combine two springs. As the equilibrium points were scarcely ever at the quadrants, they were reduced to them by interpolation, and tables formed with first and second differences, which easily gave T the tension in grains corresponding to a given θ.*

(27.) Now things being thus arranged, if the whirling machine be put in action the anemometer will turn and tend the spring till its elastic force balances the pressure $\alpha V'^2$; when this has occurred I place on the brake a weight of 6 lb., which produces a friction far surpassing the tension; the chronograph is then made to record N, generally for three minutes. The machine is then stopped, the brake holds the anemometer immovable, and θ is read off on the circle to a tenth of a division. But the tensions thus obtained are too large: the anemometer's motion is *accelerated* up to the point where $\alpha V'^2 = T + F$; there it has acquired a velocity which carries it on beyond this. The force which brings it back is now $T - \alpha V'^2 - F$, and it will rest when $\alpha V'^2 = T - F$.† Even the α thus obtained must be a trifle too large, for V′ ought to include, as a component, the radial vortex-motion Z. I subjoin the measures which I have taken in the following tables. Of the headings, D, S, N, b, V, W, and f''' have been explained; C is the reading of the circle, T the corresponding tension in grains, θ the angle made by the highest of the cups with the horizon.

* It seemed unnecessary to give these tables.

† This supposes that the moment due to the final velocity is greater than 2F, which is a minor limit of α. I have used it as giving results nearest to those obtained by minimum squares. If the moment does not exceed F, $\alpha = \frac{T+F}{V'^2}$.

Table VI.

No. I. f=113·3 ; two springs ; June 20, Bar. =29·77 ; Therm. =73·5.*

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	a.
I.	17	180	31·25	0	6·75	0	5·4	16·5	564	59°·4	9·749
II.	18	,,	39·25	0	8·48	0	10·2	21·8	638·6	78·5	7·145
III.	19	,,	42·5	214	9·18	0·61	12·5	38·5	1035·5	48·6	12·359
IV.	20	,,	46·75	214	10·10	0·61	14·6	40	1068·6	54·0	10·422
V.	21	,,	50·6	198·6	10·93	0·66	17·13	42	1155·4	61·2	9·917
VI.	22	,,	55·7	161·6	12·03	0·81	20·9	66·5	1616·7	55·8	11·793
VII.	23	,,	58·5	162·4	12·64	0·80	23·2	66·5	1616·7	55·8	10·394
VIII.	25	,,	66·5	128·8	14·37	1·01	28·6	88·7	2171·6	53·2	12·148
IX.	27	,,	68	116·6	14·69	1·12	31·8	101·2	2650·9	85·9	14·868
X.	29	,,	72·5	100	15·66	1·30	36·4	102·2	2694·7	82·1	12·327
XI.	32	,,	79	88·6	17·03	1·47	43·55	115·1	3236·1	54·4	12·697
XII.	35	,,	82·5	87·6	17·82	1·485	48·1	129·5	3712·3	75·8	13·276
XIII.	38	,,	89·3	85·2	19·29	1·53	57·15	145	4041·6	72	12·302
XIV.	41	,,	93·5	86·1	20·21	1·51	64·15	169	4543·0	68·4	12·461
XV.	44	,,	97	94·8	20·96	1·37	69·2	190	5156·2	54	12·978
XVI.	48	,,	100·5	72·3	21·71	1·80	74·2	213	6012·6	46·8	14·669
XVII.	52	,,	108·8	45·1	23·50	2·00	88·79	215	6090·9	54	12·714

The mean of the 17=11·896. They differ more than might be expected, considering that V and T are pretty certain. The discrepancies evidently are connected with the fluctuations of W and F, and are a sort of measure of their uncertainty. It is also evident that they follow no law which might indicate the presence of any power of V′ in the equation (I.), except the squares; the values of θ also show the effect of the disturbing influences. If, as is probable, the mean value of θ is that at which a is a maximum, it is here 62°·3, and the a of No. V. should be the largest, the other should decrease as their θ's recede from that value ; but nothing of the sort is observable.

Table VII.

No. II. f=101·3 ; June 15, Bar. =29·63 ; Therm. =62.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	a.
I.	14	180	47	204·6	10·15	0·64	11·6	20·7	245·3	72°·5	1·456
II.	15	,,	55·2	214·3	11·48	0·61	15·0	30	358·7	72·0	2·045
III.	18	,,	72	152·4	15·555	0·85	28·15	50	612·3	90·0	2·229
IV.	21	,,	83·2	115	16·88	1·13	33·4	53·5	658·2	77·4	2·104
V.	24	,,	95·8	91·4	20·15	1·42	48·9	96	962·0	75·6	2·310
VI.	27	,,	106	86·8	22·90	1·70	64·7	121·5	1363·3	77·4	2·657
bis.								128	1428·7	79·2	2·803
VII.	30	,,	114·75	81	24·79	1·61	77·2	129	1460·8	75·6	2·379
bis.								144·5	1596·4	70·2	2·631
VIII.	33	,,	121·5	80	26·25	1·63	87·8	146	1607·2	75·6	2·333
IX.	36	,,	138·5	64·6	29·92	2·01	118·5	195	2093·7	72·0	2·400
X.	39	,,	147	57·8	31·76	2·25	140·3	196	2103·7	75·6	2·132
XI.	45	,,	158·5	63·6	34·34	2·05	164·5	269	3003·0	68·4	2·634

The mean of the 13=2·316. Here also there is no appearance of the equation

* The coefficients α, β, and γ depend on the air's density. This must be allowed for in reducing the equations.

containing any power of V but the square. If we reduce the mean α of No. I. and No. III. in the ratio of the areas $\frac{16}{81}$ and the air's density, we get 2·477, so nearly that actually found that we may fairly assume the α to be proportional to the areas of the cups.

Table VIII.

No. III. f=201·3; June 19, Bar. 29·83; Therm. 67.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	α.
I.	18	180	52·5	120·4	11·34	1·08	17·4	67·25	1326·6	62·1°	10·196
II.	21	,,	58·6	118·4	12·675	1·10	22·0	92	1918·2	61·2	12·625
III.	24	,,	68	101	14·69	1·29	29·9	107·5	2293·9	63·0	11·466
IV.	27	,,	74·4	82·2	16·07	1·58	36·2	142	3118·1	64·2	13·385
V.	30	,,	81·3	93·2	17·56	1·40	43·8	158	3477·5	61·2	12·063
VI.	33	,,	84·5	68·8	18·255	1·89	47·6	167	3650·2	61·2	12·393
VII.	36	,,	91·6	63·4	19·70	2·05	56·2	188	4178·2	45·2	12·571
VIII.	39	,,	99	58·6	21·39	2·22	67·4	201	4509·4	89·4	11·521
IX.	42	,,	103·7	59·6	22·40	2·18	74·7	221	5074·5	75·6	11·714
X.	45	,,	107·5	64·2	23·06	2·03	79·7	248	5482·3	82·8	11·843
XI.	48	,,	115	51·4	24·84	2·53	94·4	341	6828·0	57·6	13·098

The mean of the eleven =12·107; that of No. I. reduced to the same air density =12·383. Making reasonable allowance for errors, I think we may infer that the α is not changed by reducing the arms of the anemometer to half their length.

(28.) I tried similar measures with No. IV., but had very indifferent success. The main cause of this was the bad quality of a supply of oil which had just been sent in for lubricating the machinery, and which probably prevented the excursion of the anemometer beyond the point of balanced tension. From the high temperature, also, it was necessary to have all the windows open. Eleven were taken, of which from the second to the seventh, C changed only from 23 to 25·5, though V varied from 18·1 to 28·5. C then passed at once to 94, and continued increasing. The extreme hypothesis would make $\alpha=\frac{F+T}{V'^2}$; it is precarious, but I give those that are not palpably wrong for what they are worth.

Table IX.

No. IV. f=185·0; June 21, Bar. 29·66; Therm. 75·0.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	α.
I.	15	180	66	361	14·26	0·36	20·2	20	364·8	72·0°	2·940
II.	17	,,	84	153	18·15	0·85	33·6	23	426·9	82·8	2·147
VIII.	33	,,	141	117·4	30·46	1·11	105·3	94	1967·7	68·4	2·614
IX.	34	,,	150	70·8	32·41	1·84	121·5	96	2017·5	75·6	2·480
X.	39	,,	159	91·6	34·35	1·42	129·4	121	2594·8	89·4	2·677
XI.	42	,,	159	72·3	34·35	1·80	129·4	97	2042·8	79·2	2·168

The mean of the six=2·502. The α of No. II. reduced to the present density =2·288. The irregularity of W is notable, as also that XI. with a larger D has only the same V as X. As it was at this period necessary to prepare for the erection of the Vienna equatorial, I could not repeat these experiments, but the results obtained with Nos. I. and III. leave no room for doubting that a similar agreement would be found here with No. II.

(29.) No. V. was not tried this way. The less exact method of gradually increasing the brake friction till the anemometer stopped, gave by two observations α=10·400 on June 2, Bar. 30·06, Therm. 71.

(30.) After these preliminary details I proceed to state the experimental results, beginning with those of No. 1. The first 79 of these were obtained while the centrifugal pressure was opposed by a single friction roller, and the second set (80 to 123) were taken after two were applied.

Most of the headings of the tables have been already explained; $m=\frac{V-W}{v}$ the ratio of the wind's velocity to that of the anemometer; ρ is the factor to reduce the air's density when α was measured to that at each observation.

TABLE X.

No. I. f=113·2; May 8, Bar. 30·37; Therm. 52.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
1	15	240	33	39	290	5·35	1·39	0·45	6·5	0·06616	3·519
2	16	180	32·6	34·8	242	7·04	1·66	0·54	11	”	3·935
3	18	180	41·3	52	173·3	8·92	2·475	0·74	17·2	”	3·301
4	20	180	48·71	65	147·3	10·52	3·09	0·89	22·9	”	3·115
5	24	180	60·4	85·17	122·3	13·05	4·05	1·095	39·2	”	2·948
6	28	180	69	103·7	104	14·91	4·94	1·26	47·1	”	2·7655
7	33	240	103·3	160·2	96·2	16·74	5·72	1·36	77·35	”	2·689
8	39	240	119	191·5	81·8	19·28	6·84	1·59	101·6	”	2·587
9	46	240	133·9	216·2	74	21·695	7·72	1·765	138·3	”	2·582
10	52	180	106·33	181·5	67·5	22·97	8·64	1·93	158·6	”	2·435
11	58	180	113	198	63·5	24·41	9·425	2·05	184·7	”	2·372
12	66	180	123·5	216·9	58	26·68	10·32	2·25	229·95	”	2·366
13	75	240	178·5	310	53·6	28·92	11·07	2·43	296·7	”	2·393

B=3 oz.; f+B′=398·0.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
14	18	240	42·5	19·5	159·8	6·89	0·70	0·815	10·5	0·06616	9·872
15	20	240	56·58	51 67	101·6	9·17	1·845	1·28	18·9	”	4·272
16	20¾	240	59·9	64·12	110·8	9·71	2·18	1·18	21·4	”	3·955
17	23¼	240	71·67	82	95·6	11·61	2·93	1·36	32·0	”	3·502
18	25	240	84·8	114·5	91	13 74	4·09	1·43	45·9	”	3·011
19	28	240	93·5	131·4	98·8	15·15	4·69	1·32	57·8	”	2·948
20	33·2	180	80·3	111·8	84·4	17·35	5 62	1·515	79·9	”	2·8135
21	39	180	91	142·5	81·8	19 66	6·78	1·64	107·5	”	2·656
22	46	180	100·9	166·8	74	21·80	7·93	1·83	139·9	”	2·519
23	52	180	108	180	67·5	23·33	8·57	1·95	165·7	”	2 496
24	58	240	151·5	263·9	63·5	25·09	9·42	1·97	219·5	”	2·454
25	66	240	169·7	297	58	27·49	10·60	2·14	258·8	”	2·391
26	75	240	184	332·3	53·6	29·81	11·51	2·20	323·4	”	2·400

B=6 oz; f+B′=687·6; May 30, Bar.=30·20; Therm. 61.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''	Log. ρ.	m.
27	17·5	240	56·45	24·85	130	9·15	0·89	1·005	19·4	0·03873	9·179
28	18	,,	60·9	35·5	115·4	9·87	1·27	1·13	22·2	,,	6·894
29	20	,,	69·6	57·5	110	11·28	2·05	1·185	30	,,	4·916
30	25	,,	88·1	98·4	95·4	14·27	3·51	1·37	52·3	,,	3·674
31	27	,,	108	127	86·4	17·50	4·53	1·51	70·8	,,	3·527
32	38	,,	120·1	173·3	75·2	19·47	6·19	1·73	95·5	,,	2·8665
33	45	300	168	253	73·4	21·78	7·23	1·78	139·5	,,	2·768
34	52	240	148	228·9	75	23·98	8·17	1·74	178	,,	2·722
35	58	,,	158·5	254·5	64·2	25·68	9·09	2·03	214·4	,,	2·603
36	65	,,	168·5	272	61·8	27·30	9·71	2·11	276·8	,,	2·594
37	75	,,	182·5	302	58·4	29·525	10·78	2·235	316·8	,,	2·531

B=9 oz; f+B′=955·0; May 15, Bar.=30·16; Therm. 58.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
38	19·5	180	49·25	9·5	133	10·64	0·45	0·98	26·3	0·04636	21·579
39	24	240	74·75	39·9	102·6	12·11	1·42	1·27	35	,,	7·688
40	28	,,	92·9	85·4	90·8	15·05	3·05	1·44	56·9	,,	4·465
41	31	,,	99·25	111	85·2	16·08	3·96	1·53	66	,,	3·6715
42	42	,,	126	175·3	70·6	20·42	6·26	1·85	118·3	,,	2·968
43	46	,,	136	192·3	77·8	22·04	6·865	1·68	143·9	,,	2·966
44	55	,,	148	227·5	65·4	23·98	8·12	1·995	178	,,	2·707
45	59	180	124·5	195·5	67·2	26·89	9·31	1·941	243·5	,,	2·682*
46	65	240	166·0	265·5	60·8	26·90	9·48	2·15	273·9	,,	2·611
47	75	,,	182·5	298·5	57·2	29·57	10·66	2·28	315·9	,,	2·561

B=12 oz.; f+B′=1211·3.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
48	25	240	82	37·6	102·6	13·29	1·34	1·27	46·9	0·05971	8·950
49	30	,,	98·4	86·2	95·6	15·95	3·08	1·365	65·9	,,	4·741
50	32	180	76·9	70·7	86·6	16·61	3·365	1·51	72·6	,,	4·488
51	40	240	120·5	140·7	74·8	19·525	4·84	1·74	105·6	,,	3·670
52	46	,,	131·7	166·2	68·4	21·34	5·93	1·91	132	,,	3·376
53	54	,,	147	203·5	68·8	23·82	7·265	1·92	174·9	,,	3·014
54	58	,,	155·1	216	65·8	25·13	7·71	1·98	202	,,	3·003
55	65	,,	166·5	249·8	66	26·98	8·92	1·98	243·5	,,	2·804
56	75	,,	183·3	281·4	62·8	29·70	10·05	2·08	319·8	,,	2·750

* At the end of this it was noted that one of the cups was disfigured by something striking it; 45 must therefore be rejected: it is given here to show how little v was affected by a considerable deformation of the hemisphere. I am not certain but the accident may have occurred earlier, the cup was immediately restored to shape, but the refixing it put it a little out of balance, which was not corrected till the 16th. Some dirt also had got to the outer friction wheels, so that Nos. 46 and 47 and 73 to 79 were not of equal weight with the others.

B=15 oz.; f+B′=1460·9; May 12, Bar.=30·12; Therm. 53.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f‴*.	Log. ρ.	*m*.
57	28	240	92·8	34·2	93	15·04	1·23	1·40	56·7	0·05971	11·095
58	31	”	98·4	57·9	85·2	15·95	2·07	1·53	65·3	”	6·975
59	39	”	119·4	124·5	76·8	19·35	4·445	1·70	103·3	”	3·971
60	45	”	133·75	160	69	21·67	5·71	1·89	137·8	”	3·463
61	52	”	148·25	193	70·6	24·02	6·96	1·85	178·8	”	3·185
62	56	”	152·8	207·4	64	24·76	7·405	2·04	194·1	”	3·068
63	64	”	165·45	234	60·6	26·805	8·35	2·53	241·3	”	2·951
64	75	”	181·2	267·5	60·2	29·36	9·55	2·17	309·4	”	2·8475

B=18 oz.; f+B′=1707·9; May 13, Bar.=30·34; Therm. 55.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f‴*.	Log. ρ.	*m*.
65	28	180	72·7	33·3	97	14·00	1·585	1·345	48	0·05725	7·9825
66	31	240	100	56·2	91·9	16·20	2·01	1·42	67·8	”	7·367
67	39	”	120	105·5	81·4	19·44	3·77	1·60	104·5	”	4·737
68	45	”	131·5	140·75	68·8	21·31	5·025	1·92	131	”	3·8575
69	54	”	147·75	187	67·1	24·26	6.68	1·94	183·8	”	3·343
70	58	”	156·25	202·75	64·6	25·32	7·24	2·02	206·2	”	3·219
71	65	”	166·75	226	60	27·02	8.07	2·17	246·5	”	3·079
72	75	”	183	265·75	59·6	29·65	9.49	2·19	318	”	2·895

B=32 oz.; f+B′=2827·9; May 15, Bar.=30·16; Therm. 58.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f‴*.	Log. ρ.	*m*.
73	32	240	102·4	36	96	16·59	1·285	1·359	71	0·04636	11·853
74	41	”	122·4	80	72·2	19·83	2·85	1·81	109·8	”	6·311
75	56	”	136	117	66·4	22·03	4·18	1·96	143·8	”	4·804
76	?	”	158·8	161	62·8	25·73	5·85	2·09	215·2	”	4·037
77	60	”	160	180	62·2	25·92	6·43	2·10	220	”	3·708
78	66	”	169	200	63·2	27·38	7·14	2·07	254·4	”	3·545
79	75	”	185	244	61·4	29·975	8·71	2·125	328	”	3·197

A change was now made in the friction rollers, in consequence of which it seemed desirable to take another series of No. 1.

B=0; f=113·2; May 24, Bar. 29·702; Therm. 60.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f‴*.	Log. ρ.	*m*.
80	15	240	37	38	294·4	5·99	1·36	0·45	5·8	0·03423	4·088
81	18	”	61	81·9	148·2	9·88	2·92	0·88	9·9	”	3·3805
82	22	”	79·5	116·5	107·2	12·88	4·16	1·22	16·8	”	2·806
83	28	”	98	151	83·8	15·88	5·50	1·56	27·4	”	2·605
84	35	”	112	185	83·8	18·15	6·60	1·56	37	”	2·512
85	45	”	133·5	225	69·8	21·63	8·03	1·87	57·4	”	2·461
86	56	”	153	265	59·2	24·79	9·46	2·20	81·4	”	2·387
87	65	”	169·2	294·8	55·6	27·41	10·52	2·345	106·9	”	2·383
88	75	”	184	324	56	29·81	11·84	2·33	134·5	”	2·322
89	85	”	200	354	54·8	32·41	12·64	2·38	171	”	2·375
90	.105	”	224	399·6	53·4	36·29	14·27	2·44	238	0·05728	2·373

B=6 oz.; f+B′=687·6; May 25, Bar. 29·90; Therm. 56·1.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
91	17	240	55·9	22·7	173·6	9·06	0·81	0·75	7·8	0·05625	10·251
92	22	"	80	71·75	111	12·96	2·56	1·17	17	"	4·602
93	28	"	100	114·25	87·8	16·20	4·08	1·485	27·9	"	3·609
94	33	"	110	137·6	86·4	17·32	4·91	1·51	35·4	"	3·3205
95	43	"	132·8	187	72·2	21·52	6·68	1·81	56·7	"	2·952
96	54	"	153·5	234	63·4	24·87	8·35	2·06	82·1	"	2·731
97	63	"	166	264	56·6	26·90	9·425	2·30	101·5	"	2·609
98	74	"	182	294	56	29·49	10·50	2·33	131·0	"	2·588

B=12 oz.; f+B′=1211·3.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
99	23	240	76	26	174	12·31	0·93	0·75	15·4	0·05625	12·460
100	29	"	97·7	63	92·4	15·83	2·25	1·41	27·2	"	6·411
101	34	"	109	92	83	17·66	3·28	1·57	34·7	"	4·899
102	44	"	136	151	73·8	19·44	5·39	1·77	43·9	"	3·278
103	55	"	153	204	62·2	24·79	7·23	2·09	81·4	"	3·117
104	64	"	171	236	58·4	27·71	8·425	2·235	110·1	"	3·023
105	74	"	184	272	53·4	29·81	9·71	2·44	134·6	"	2·8185

B=18 oz.; f+B′=1707·9; May 26, Bar. 29·84; Therm. 58·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
106	29	240	94	35	85·6	15·23	1·25	1·52	24·8	0·04113	10·970
107	33	"	108	52	85·6	17·50	1·61	1·52	34·4	"	9·941
108	44	"	136	116	73	22·04	4·14	1·79	60·3	"	4·890
109	55	"	154	163	63·2	24·95	5·82	2·06	82·8	"	3·933
110	64	"	168	204	60·4	27·22	7·28	2·16	105	"	3·442
111	74	"	184	242	55·6	29·81	8·64	2·32	134·5	"	3·182

B=24 oz.; f+B′=2140·1.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
112	34	240	111	42	91·6	17·985	1·50	1·425	36·2	0·05625	11·090
113	44	"	136	84·2	72	22·04	3·01	1·81	60·3	"	6·728
114	55	"	156	140	63·4	25·28	5·00	2·06	85·8	"	4·646
115	64	"	169	179	61	27·38	6·39	2·14	91·5	"	3·950
116	74	"	185	220	57·6	29·975	7·85	2·27	136·6	"	3·529

B=30 oz.; f+B′=2669·3; May 26, Bar. 29·84; Therm. 58·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f‴.	Log. ρ.	m.
117	44	240	131	40	73·2	21·23	1·43	1·78	54·8	0·04113	13·617
118	55	"	152	84	62	24·63	3·00	2·10	79·9	"	7·5105
119	64	"	165·5	129·2	62·8	26·815	4·61	2·075	100·7	"	5·634
120	74	"	180	175	56·6	29·165	6·25	2·30	126·9	"	4·299

B=36 oz.; f+B′=3144·4.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*′′′.	Log. ρ.	*m*.
121	55	240	148	70·4	67·4	23·98	2·51	1·92	79·5	0·04113	8·774
122	64	,,	165	112	63	26·73	4·00	2·17	99·9	,,	6·155
123	75	,,	180·5	152	57·8	29·25	5·43	2·26	128·4	,,	4·974

Table XI.

No. II. f=101·3; June 6, Bar. 29·695; Therm. 62.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*′′′.	Log. ρ.	*m*.
124	12½	420	113·4	55·4	241·6	10·50	1·13	0·54	8·2	0·00095	8·813
125	15	240	92	71	184	14·91	2·535	0·82	18·1	,,	5·518
126	18	,,	113·0	99·5	156·2	18·31	3·55	0·97	31·3	,,	4·910
127	22	,,	133	131	115·2	22·52	4·75	1·31	47·7	,,	4·415
128	25	,,	152	156	102·2	24·628	5·57	1·48	61·7	,,	4·157
129	28	,,	164	166·0	104·4	26·57	5·93	1·45	72·3	,,	4·266
130	35	,,	195·2	204	82	31·27	7·28	1·84	112·0	,,	4·090
131	40	,,	217	237	74	35·16	8·46	2·045	155·4	,,	3·914
132	48	,,	240·8	276·7	68·4	39·02	9·84	2·24	210·1	,,	3·739
133	56	180	196	232·2	67·2	42·34	11·05	2·25	268·7	,,	3·627

B=1 oz.; f+B′=167·0; June 7, Bar. 29·895; Therm. 63.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*′′′.	Log. ρ.	*m*.
134	18½	180	106	80	138·6	17·175	2·86	0·94	25·2	9·99883	5·685
135	23	,,	135·5	125¼	111·2	21·955	4·47	1·04	44·8	,,	4·678
136	26	,,	152·6	148¼	102·2	24·725	5·30	1·27	59·9	,,	4·432
137	29	,,	158	157	94·4	25·60	5·605	1·38	65·65	,,	4·322
138	36	,,	185¾	192¼	75	30·10	6·83	1·73	100·8	,,	4·154
139	41	,,	156	168·5	65	34·03	8·02	2·00	141·4	,,	3·992
140	48	120	115·5	129·5	64·2	37·43	9·24	2·02	236·7	,,	3·833

B=2 oz.; f+B′=280·4.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*′′′.	Log. ρ.	*m*.
141	23	240	138	110¾	104	22·36	3·95	1·25	49·75	9·99883	5·339
142	26	,,	161·3	146·3	81·2	26·13	5·22	1·60	69·2	,,	4·697
143	29	,,	166	155	87	26·90	5·53	1·495	74·75	,,	4·554
144	36	,,	205	214·8	72·6	33·215	7·67	1·795	132·05	,,	4·097
145	41	,,	223·2	241	62·6	36·16	8·60	2·08	168·6	,,	3·9615
146	48	180	186	213·5	56·4	40·18	10·16	2·30	233·1	,,	3·727

B=3 oz.; f+B′=398·0.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*′′′.	Log. ρ.	*m*.
147	26	246	164·5	130¼	82	26·65	4·44	1·59	73·0	9·99883	5·6315
148	29	,,	175	149	74	28·35	5·32	1·76	85·7	,,	5·000
149	36	,,	206·2	200	70·4	33·41	7·14	1·85	134·3	,,	4·420
150	41	,,	225·5	233·5	62·6	36·54	8·34	2·08	173·7	,,	4·136
151	48	180	187	203·7	51·6	40·40	9·70	2·52	232·8	,,	3·9065

B=4 oz.; f+B′=486·2; June 8, Bar. 29·745; Therm. 59·5.

No.	D.	S.	N.	A.	*b.*	V.	*v.*	W.	f'''.	Log. ρ.	*m.*
152	29	240	175·9	132	81·6	28·50	4·71	1·59	86·9	9·99156	5·709
153	36	„	203	187·3	60·2	32·88	6·69	2·16	128·4	„	4·595
154	41	„	223	214	58	36·13	7·64	2·24	167·8	„	4·436
155	48	„	245·9	253	55·8	39·835	9·03	2·33	224·9	„	4·042

B=5 oz.; f+B′=582·5.

No.	D.	S.	N.	A.	*b.*	V.	*v.*	W.	f'''.	Log. ρ.	*m.*
156	36	240	202·4	163·4	61·4	32·79	5·85	2·12	127·3	9·99156	5·246
157	41	„	221	199·8	63·6	35·80	7·14	2·045	163·7	„	4·732
158	48	180	182	181·5	58·2	39·32	8·64	2·235	215·0	„	4·292

No. III. was originally constructed with arms of steel plate $\frac{1}{8}$th of an inch thick, as I thought that with a length of 12 inches there would not be much danger of flexure. Sixteen observations were made with it in this state; but during the seventeenth, with D=75, the screw which attached one of the arms to the centre piece snapped off, though 0·32 inch thick. The arms were now made of steel tube similar to No. I. On June 8 an observation, 164, was made with D=35, which gave exactly the V of 163; and a *v* only 0·071 less. At the time I thought this difference was merely casual, and did not repeat these sixteen observations; but it is more probable that the difference is real, in part at least, and must be kept in mind when discussing the observations. These are given in Table XII., No. III.

TABLE XII.

No. III f=201·3; May 30, Bar. 29·94; Therm. 67·9.

No.	D.	S.	N.	A.	*b.*	V.	*v.*	W.	f'''.	Log. ρ.	*m.*
159	14·5	240	35$\frac{3}{4}$	60	283·2	5·79	1·07	0·48	5·9	9·99927	4·964
160	18	„	64·9	179·5	111·4	10·515	3·20	1·23	20·4	„	2·896
161	22	„	83	246	85·4	13·45	4·39	1·59	34·6	„	2·699
162	28	„	101	312	69·7	16·36	5·57	1·95	54·5	„	2·589
163	35	„	120	392	66·2	19·44	7·00	2·05	81·6	„	2·4855
164	35	„	120	388	66·4	19·44	6·93	1·89	81·9	„	2·534
165	45	„	145·7	482·8	56	23·61	8·62	2·42	130·7	„	2·461
166	56	180	124	424	56	26·79	10·09	2·42	184	„	2·415
167	75	240	195·2	676	61	31·63	12·07	2·23	290·8	„	2·4365

B=3 oz.; f+B′=770·8.

No.	D.	S.	N.	A.	*b.*	V.	*v.*	W.	f'''.	Log. ρ.	*m.*
168	17·5	240	65·3	44	103·4	10·58	0·77	1·34	20·7	9·99927	11·963
169	22	„	84	154·9	93	13·61	2·765	1·46	35·7	„	4·394
170	28	„	102	249·6	81·2	16·53	4·445	1·67	56	„	3·324
171	37	„	123·5	351·5	76	20·01	6·27	1·79	88	„	2·905
172	47	„	150·8	472·4	77·2	24·43	8·43	1·76	143·4	„	2·689
173	57	„	165·6	535	59·6	26·83	9·55	2·28	184·9	„	2·571
174	66	„	182·6	608·3	66·2	29·59	10·86	2·05	246	„	2·519

B=6 oz.; f+B′=1349·9; June 3, Bar. 29·56; Therm. 61·5.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
175	22	240	83·5	66·3	118	13·53	1·18	1·28	35·1	0·01060	10·245
176	28	„	102·2	165	91·2	16·56	2·945	1·66	56·25	„	5·060
177	36	„	124	279	84·8	20·09	4·98	1·78	88·8	„	3·676
178	46	„	146	393·6	65·2	23·66	7·03	2·39	134·7	„	3·028
179	56	„	164	481·4	66	26·57	8·59	2·29	180	„	2·826
180	66	„	183	570·7	59·4	29·65	10·20	2·55	245·4	„	2·6545
181	75	„	198·5	631·3	58·2	32·16	11·27	2·60	304·1	„	2·623

B=9 oz.; f+B′=1884·7.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
182	28	240	100·5	45	99·2	16·28	0·82	1·52	54·7	0·01060	18·376
183	36	„	123·5	168·5	90·2	20·01	3·01	1·68	88	„	6·096
184	47	„	148	337·5	72	23·98	6·02	2·10	136·6	„	3·633
185	57	„	167	436	66·4	27·06	7·78	2·27	189·3	„	3·185
186	66	„	183	517·7	62·8	29·65	9·24	2·41	245·4	„	2·948
187	75	„	197	585·2	56·2	31·92	10·445	2·69	297·5	„	2·798

B=12 oz.; f+B′=2397·5.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
188	36	240	126	142·3	94·4	20·42	2·54	1·65	92·4	0·01060	7·397
189	47	„	150·3	310·4	74	24·35	5·54	2·04	142·1	„	4·0265
190	57	„	166·5	369	69·4	26·98	6·59	2·18	187·8	„	3·765
191	66	„	177	431	63	28·68	7·69	2·40	225·5	„	3·416
192	76	„	198	530	57	32·08	9·46	2·65	302·1	„	3·111

B=15 oz.; f+B′=2896·5.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
193	47	240	146	195	76·4	23·66	3·48	1·98	131·5	0·01060	6·228
194	57	„	164	332·9	68·2	26·57	5·94	2·22	180	„	4·100
195	66	„	179	408	61·8	29·00	7·28	2·45	232	„	3·646
196	76	„	196	496	60·8	31·76	8·85	2·49	294	„	3·306

B=18 oz.; f+B′=3390·5.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
197	57	240	164·4	274·3	67·6	26·64	1·90	2·24	181·3	0·01060	4·984
198	66	„	181·3	384	62·2	29·375	6·85	2·43	239·5	„	3·931
199	76	180	147	339·5	56·6	31·76	8·08	2·67	293	„	3·599

The results with No. IV. are less reliable than any of the others; the shorter arms and smaller cups having less power to overcome the casual irregularities of friction.

Table XIII.

No. IV. f=185·0 ; June 21, Bar. 29·66 ; Therm. 75.*

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
200	14	240	82·7	104	184	13·40	1·86	0·71	24·3	9·96688	6·836
201	16	”	106·4	181·8	143	17·22	3·245	0·91	42·8	”	5·025
202	20	”	121·8	242·3	152	19·73	4·325	0·86	58·4	”	4·364
203	18	”	136·8	292·6	167·6	22·16	5·22	0·78	76·9	”	4·095
204*	22	”	142·8	314	111	23·14	5·60	1·17	85·1	9·97611	3·916
205	24	”	149·3	339	111	24·19	6·05	1·17	95·2	9·96688	3·831
206	21	”	155·5	359·8	106·1	25·19	6·42	1·22	104·6	9·97149	3·733
207*	26	”	166·6	392·7	96·2	26·99	7·01	1·35	125·8	9·97611	3·581
208*	28	”	175·6	425·5	82·9	28·45	7·59	1·58	143·4	9·97149	3·539
209	28	”	181·9	439	86	29·47	7·835	1·51	157·5	9·96688	3·568
210	30	”	194·2	485·8	78	31·465	8·67	1·61	187·8	”	3·437

B=1·5 oz. ; f+B′=432·0 ; June 22, Bar. 29·95 ; Therm. 73.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
211	16	240	120	212·8	178	19·485	3·80	0·73	56·6	9·97611	4·938
212	18	”	134·9	274	167·6	21·86	4·89	0·78	74·5	”	4·290
213	20	”	151	333	153·3	24·47	5·94	0·85	97·7	”	3·9775
214	22	”	162·2	370·9	93·3	26·28	6·62	1·395	117·5	”	3·761
215	24	”	178	424	82·4	28·84	7·57	1·58	148·7	”	3·603
216	25·5	”	184	444·2	82	29·81	7·89	1·59	163·4	”	3·576
217	27·5	”	196	495	78·8	31·76	8·835	1·65	192·8	”	3·408
218	29·5	”	199	506·7	69·6	32·24	9·04	1·87	201·7	”	3·359

B=3 oz. ; f+B′=754·5.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
219	18	240	125·7	99·3	133	20·365	1·77	0·98	63	9·97611	10·939
220	20	”	147·6	217	120	23·915	3·87	1·08	92·5	”	5·8945
221	22	”	160	274	97·2	25·92	4·89	1·34	113·5	”	5·027
222	24	”	172	328	92·6	27·87	5·85	1·405	135·9	”	4·520
223	26	”	187	379·6	63·2	30·30	6·775	2·06	168·9	”	4·169
224	29	”	205·7	456	79·2	33·83	8·14	1·64	227	”	3·893

B=4·5 oz. ; f+B′=1052·0 ; June 23, Bar. 29·68 ; Therm. 72.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
225	23	240	176	238	80	28·52	4·25	1·63	144·3	9·97782	6·330
226	25	”	188	304	72·6	30·46	5·43	1·795	168·8	”	5·283
227	27	”	199	342·8	67·8	32·24	6·12	1·92	200·7	”	4·956
228	29	”	208·5	394	64·2	33·785	7·03	2·03	226	”	4·516

* Those marked * were observed on June 22. There is a great confusion in the D's. This may have arisen from irregular friction of the vertical axes. See the remarks preceding Table IX.

B=6 oz.; f+B′=1333·6; June 22.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
229	23	240	170·2	108	77	27·58	1·60	1·69	132·7	9·97611	16·140
230	26·5	,,	192	224	71	31·11	4·00	1·83	182	,,	7·323
231	29·5	,,	202	267	69·8	32·89	4·77	1·86	209	,,	6·511

Table XIV.

No. V.—Cylinder Cups. f=118·5; May 31, Bar. 30·165; Therm. 67·9; Standard for ρ, Bar. 30·05; Therm. 71.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
232	15	240	40	38	196·2	6·48	1·36	0·77	4·3	0·00946	4·209
233	22	,,	65·2	77·3	112·2	11·05	2·76	1·36	13·3	,,	3·512
234	28	,,	82·3	102	73·6	13·335	3·64	2·05	19·6	,,	3·098
235	35	,,	96·8	124·5	70·1	15·68	4·445	2·16	29·4	,,	3·043
236	45	,,	114·3	151·5	72	18·52	5·41	2·10	53·1	,,	3·0345
237	56	,,	129·3	173	65	20·95	6·18	2·33	59	,,	3·0155
238	65	,,	141	190	57·4	22·85	6·78	2·64	73·3	,,	2·979
239	75	,,	152	208	57	24·63	7·43	2·65	89·7	,,	2·9595

B=6 oz.; f+B′=692·8; June 1, Bar. 30·17; Therm. 70.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
240	25	240	79	44·9	87·8	12·80	1·60	1·72	17·7	0·00442	6·911
241	37	,,	102	97	87	16·53	3·46	1·74	33·5	,,	4·271
242	46	,,	117·2	124·9	65	18·99	4·46	2·33	46	,,	3·737
243	57	,,	132	150	61·6	21·39	5·355	2·46	62·2	,,	3·535
244	66	,,	143	169	53·8	23·17	6·03	2·81	76·1	,,	3·375
245	76	,,	155·5	189·2	50·1	25·195	6·75	3·02	95·5	,,	3·283

B=12 oz.; f+B′=1216·6.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
246	35	240	98	41·5	72·8	15·88	1·48	2·08	30·3	0·00442	9·316
247	46	,,	117·4	89·3	63·8	19·02	3·19	2·40	46·2	,,	5·215
248	57	,,	132	121	59·8	21·39	4·355	2·53	62·2	,,	4·330
249	66	,,	144	144	56·8	23·33	5·14	2·64	77·5	,,	4·024
250	76	,,	157	164·5	51·6	25·46	5·87	2·93	98·3	,,	3·836

B=18 oz.; f+B′=1715·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
251	46	240	119	49	64·4	19·28	1·75	2·34	47·8	0·00442	9·685
252	56	,,	132	98	59·6	21·39	3·23	2·54	52·2	,,	5·387
253	66	,,	144	125	53·6	23·33	4·46	2·82	77·6	,,	4·596
254	76	,,	156·3	146·6	55·2	25·32	5·23	2·74	96·8	,,	4·315

$B=24$ oz.; $f+B'=2145{\cdot}4$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
255	56	240	131	69	61	21·23	2·46	2·46	60·9	0·00442	7·618
256	66	„	144	105	55·4	23·33	3·75	2·73	77·6	„	5·763
257	76	„	156·8	130·5	53	25·405	4·66	2·85	97·7	„	4·837

On examining these tables it is seen that m, the ratio of the wind's velocity to that of the anemometer, is different in each of the four instruments, and varies in each. In No. I. the extreme values are 21·58 and 2·32; in No. II. 8·81 and 3·63; in No. III. 18·00 and 2·41; in No. IV. 16·14 and 3·44.* It decreases with the increased size of the cups and length of the arms, and with increased v it increases with F. On examining its rate of decrease with an increasing v it is seen to be of an asymptotic character, such that it may be expected to remain finite even when v is quasi infinite. All these conditions are satisfied by equation (I.), the positive root of which gives $\frac{V}{v}=\frac{\beta}{\alpha}+\sqrt{\frac{\beta^2}{\alpha^2}+\frac{\gamma}{\alpha}+\frac{F}{\rho\alpha v^2}}=x+\sqrt{z+\frac{F}{\rho\alpha v^2}}$ (III.).

If, as is probable, β and γ are proportional to α, the variation of m depends upon $\frac{F}{\alpha v^2}$ alone; and its limiting value$=x+\sqrt{z}$. It follows from this that the variation of m will be less in proportion as F is less, the cups larger, and the arms longer.

(31.) We can now proceed to determine the constants α, β, and γ, as already indicated, by combining the equations given by the observations by minimum squares. The present case, however, is not favourable for the employment of this method, which supposes that the error of each equation, besides being small, depends solely on errors of the constants, and that their coefficients are exact. This is by no means true here. V′ and v are both liable to errors which are variable, and F to variable ones which may be of considerable amount, and also to casual ones even larger. Still, the result so obtained is probably better than what would be obtained by dividing the equations into three groups and proceeding by elimination.

(32.) I put the equations into the form (II.),

$$\alpha-2\beta\times\frac{v}{V'^1}-\gamma\times\frac{v^2}{V'^2}=\frac{F}{\rho V'^2}\text{; or, }\alpha-2\beta\xi-\gamma\xi^2=\eta\text{ (IV.),}$$

both to avoid the large numbers that would occur in dealing with the original form (I.) and to diminish the influence of errors of V′. The measures of α, given in Tables VI., VII., and VIII., show that (I.) contains no term of V′; and in the first instance I tried one ζv, but found that it led to values of V′ so much astray that the presence of it is inadmissible.

(33.) As it was possible that α, &c., might be functions of v, I divided the equations

* These numbers explain my assuming in my original paper that the limiting value of m is 3. The anemometer with which I experimented had 3-inch cups, 6-inch arms, and considerable friction.

of No. III. (with which I commenced operations) into three groups; the first those in which $v<5$; the second those where it is between 5 and 9; the third those above 9. The final equations are:—

$$\begin{array}{llr}
1. & \alpha\times 13 -2\beta\times 2{\cdot}501-\gamma\times 0{\cdot}591= 71{\cdot}641 & \alpha= 12{\cdot}117 \\
 & \alpha\times 2{\cdot}501-2\beta\times 0{\cdot}591-\gamma\times 0{\cdot}160= 11{\cdot}226 \text{ giving} & \beta= 24{\cdot}436 \\
 & \alpha\times 0{\cdot}591-2\beta\times 0{\cdot}160-\gamma\times 0{\cdot}047= 2{\cdot}217 & \gamma=-59{\cdot}672
\end{array}$$

$$\begin{array}{llr}
2. & \alpha\times 13 -2\beta\times 3{\cdot}831-\gamma\times 1{\cdot}170= 49{\cdot}175 & \alpha= 9{\cdot}593 \\
 & \alpha\times 3{\cdot}831-2\beta\times 1{\cdot}170-\gamma\times 0{\cdot}368= 13{\cdot}451 & \beta= 6{\cdot}568 \\
 & \alpha\times 1{\cdot}170-2\beta\times 0{\cdot}368-\gamma\times 0{\cdot}021= 3{\cdot}782 & \gamma= 21{\cdot}554
\end{array}$$

$$\begin{array}{llr}
3. & \alpha\times 14 -2\beta\times 5{\cdot}077-\gamma\times 1{\cdot}864= 29{\cdot}319 & \alpha= 11{\cdot}975 \\
 & \alpha\times 5{\cdot}077-2\beta\times 1{\cdot}864-\gamma\times 0{\cdot}692= 10{\cdot}037 & \beta= 14{\cdot}287 \\
 & \alpha\times 1{\cdot}864-2\beta\times 0{\cdot}692-\gamma\times 0{\cdot}259= 3{\cdot}485 & \gamma=- 3{\cdot}617
\end{array}$$

$$\begin{array}{llr}
\text{All.} & \alpha\times 40 -2\beta\times 11{\cdot}409-\gamma\times 3{\cdot}625=150{\cdot}135 & \alpha= 9{\cdot}472 \\
 & \alpha\times 11{\cdot}409-2\beta\times 3{\cdot}625-\gamma\times 1{\cdot}220= 34{\cdot}714 & \beta= 8{\cdot}469 \\
 & \alpha\times 3{\cdot}625-2\beta\times 1{\cdot}220-\gamma\times 0{\cdot}427= 9{\cdot}484 & \gamma= 9{\cdot}814
\end{array}$$

There is here no evidence of dependence on v; but there is very great discordance. The values of α are most consistent, and do not differ much from my measures of it, being (except in one number) a little less, as I had expected. β is more aberrant; but the range of γ is extravagant: its value in 1 is unreal, for it cannot have a negative value greater than α. In the case of this set, one cause of error is obvious. When v is small, the moment of the anemometer is not sufficient to master these casual irregularities of friction, of which I have already spoken; but at higher speeds their effect is much less sensible. Another is the smallness of the coefficients of β and γ compared to those of α. In 1 the coefficients of β and γ compared to those of α are 0·38 and 0·045 that of α; in 2 are 0·59 and 0·09; in 3 are 0·72 and 0·13. We may therefore expect α to be better determined than β, and β than γ.

To make this clearer by an example. If we determine the α and γ of the first group, keeping the independent terms of the three equation as symbols, we have—

$$\alpha=\text{H}\times 2{\cdot}46-\text{H}'\times 26{\cdot}01+\text{H}''\times 57{\cdot}59,$$
$$\gamma=-\text{H}\times 55{\cdot}79+\text{H}'\times 659{\cdot}19-\text{H}''\times 1564{\cdot}41,$$

from which it is evident that errors in H will affect γ very largely in comparison of α. If we suppose the probable errors of the frictions in the observations to equal those given by my friction measures at Rathmines, we can compute those of α and γ due to *this* cause of error. In three of the thirteen observations the PE of $\text{F}=\pm 5{\cdot}4$; in three, $\pm 35{\cdot}4$; in three, $\pm 9{\cdot}7$; in two, $\pm 17{\cdot}8$; in one, $\pm 47{\cdot}0$; and in the last, $\pm 3{\cdot}2$.

The errors must have the same sign in each H. Hence I find PE of $\alpha = \pm 1{\cdot}30$; PE of $\gamma = \mp 39{\cdot}06$; thirty times that of α. The errors of W must also be injurious; the more so as they affect every term of the three equations, but it is impossible to measure them, and if known the calculation of their influence would be very complicated.

(34.) We cannot test surely the accuracy of these constants by trying which of them gives the best value of V′, for it is to be noted that they, and especially γ, may be changed to a considerable extent without ceasing to satisfy, at least approximately, the original equations.* Supposing F unchanged, we have for one of them $m^2\Delta\alpha - 2\Delta\beta \times m - \Delta\gamma = 0$. Combining this with a second one we obtain $\Delta\alpha\left(\frac{m+m'}{2}\right) = \Delta\beta$; $\Delta\gamma = -\Delta\alpha \times mm'$ (V.); and the constants so increased will satisfy these two equations. But it will be found that, except v is very small, $\frac{m+m'}{2}$ and mm' do not differ much for any other pair; and therefore if we take their means for an entire set we shall be able to find the β and γ belonging to a small change of α. Twenty pair from No. I., and as many from No. III., give $\Delta\beta = \Delta\alpha \times 3{\cdot}293$; $\Delta\gamma = -\Delta\alpha \times 11{\cdot}463$.†

(35.) As γ seems the chief difficulty, if it could be found *à priori* the others would be much better determined; but no means of doing this has occurred to me. By causing No. I. to revolve in quiescent air, convexes foremost, it was found that the resistance $= v^2 \times 28$; but this coefficient is far greater than what it would be in a current of air. When moving down that current, whose velocity is always more than $2v$, the convexes experience hardly any pressure, when against it, the pressure is largely included in the expression of α' $(V^2 + v^2)$. Therefore γ must be much less than $28 - \alpha$. We may, however, consider α as tolerably determined by the process described in paragraph 27.

(36.) In this state of uncertainty I consulted Professor STOKES, and his reply was so instructive that (with his permission) I annex it in an Appendix.

In it he suggested that as the equation (IV.) has only two variables ξ and η, it could be plotted on a plane surface; that such plotting might give valuable information, not only as to the existence of the errors which I suppose to affect the V, &c., but as to the equation (I.) really representing the conditions of the anemometer

* This is well shown in the Appendix.

† The same may be done without changing α by making $\Delta\gamma = -2\Delta\beta \times$ mean m. I tried this in rather an extreme case on the constants given in paragraph 38, changing them so that $y_1 = -x_1^2$. I got a value for x_1 identical with that obtained on this hypothesis from thirty-three of the equations in Table XII. It may be remarked that this supposition gives a formula analogous to M. DOHRANDT'S $V' = vx_1 + \sqrt{\frac{F}{\alpha}}$. This, of course, gives good results for many of the observations, but is astray both for high and low values of v. Its maximum $\Delta V' = +5{\cdot}56$; its minimum $= \div 2{\cdot}37$; while those of Table XVI. are $+2{\cdot}68$ and $-1{\cdot}17$. Its probable error is twice that of (III.).

motion, and possibly some information as to the value of constants. In the parabola of equation (IV.) its ordinate at the origin of $\xi=\alpha$; its tangent there $=-2\beta$; where the curve meets the axis of ξ the $\xi=\lambda$ is the limiting value of $\frac{v}{V'}$ and the tangent there $=-2(\beta+\lambda\gamma)$.

Plate 68 shows the plotting of No. III., in which ξ is on a scale twenty times that of η to prevent crowding; the dots are marked with numbers expressing the series to which they belong. The irregularity of their distribution is notable: least so at the higher values of ξ, where for a considerable distance from the limit their general direction is nearly a right line, which would be represented by $\eta=\alpha'-2\beta'\xi$. A slight curvature downwards may, however, be traced, indicating a γ of small magnitude. Near the origin of ξ the dots ramble so much that no curve can be drawn through them with any certainty; but their predominant tendency is in favour of a downward curvature. Four of them are so far above the right line as to explain fully the enormous negative value of γ derived from the first set of equations. It is evident that this right line $\eta=\alpha'-2\beta'\xi$ or its primitive $\alpha'V'^2-2\beta'V'v=F$ must very nearly satisfy the observations; and, at least for the higher speeds, would suffice to give V′ in terms of v and F. This supposes that the coefficient of that part of the resistance which depends on v^2 is $=0$, a supposition by no means improbable.

(37.) Assuming $\gamma=0$, the second and third of each of these equations—

1. $\alpha'=9{\cdot}999$	2. $\alpha'=11{\cdot}308$	3. $\alpha'=11{\cdot}550$	All. $\alpha'=9{\cdot}989$
$\beta'=11{\cdot}645$	$\beta'=12{\cdot}768$	$\beta'=13{\cdot}037$	$\beta'=10{\cdot}928$

These are much more consistent than the results when γ is also sought. It is obvious that nine or ten of the aberrant dots can in no wise contribute to a correct determination of α and β, and therefore that observations when ξ is low and η large had better be omitted in the minimum squares. And lastly, it may be inferred from this plotting that even if we had accurate values of α, β, and γ, we cannot expect to get very correct values of V′.

(38.) These considerations induce me to assume my measure of α, or rather 0·9 of it to be the true one, and to substitute it in the second equation.

This reduction of it is arbitrary, but I have given reasons for thinking my measures a little too large, and the general tendency of No. III. and No. I. is to give it even smaller. If on comparing V′ with that given by the constants thus obtained, it is found systematically erroneous, they may be corrected by (V.), combined with $\frac{dV}{v}=\frac{-m^2d\alpha+2md\beta+dy}{2(\alpha m-\beta)}$ (VI.).

I also reject the observations where $v<5$ as unlikely to give reliable results. I find from $v=5$ to $v=9$; $\alpha=10{\cdot}896$; $\beta=11{\cdot}360$; $\gamma=5{\cdot}176$.*

* The first and third are deduced from values of α' and β' by the formula V.; I consider them better than the direct value.

$v=5$ to $v=9$,	$\alpha=10{\cdot}896$;	$\beta=11{\cdot}360$;	$\gamma=5{\cdot}176$
All above 9 .	10·896	11·227	5·291
All above 5 .	10·896	11·830	2·437
All	10·896	13·104	−6·505
	10·896	11·880	+1·600

The values of $\frac{\beta}{\alpha}=x=1{\cdot}0903$; $\frac{\gamma}{\alpha}=y=0{\cdot}1468$; and $x^2+y=z=1{\cdot}3356$.

(39.) The observations with No. I. constitute two series: the first (1·79) taken while there was only one roller to sustain the centrifugal pressure; the other (80·123) after two were applied.

Plate 66 shows the plotting of the two: those of the first being represented by dots, those of the second by small circles. There is the same irregular distribution, and it is still more difficult to trace the curvature. It is to be remarked that the system of small circles falls *below* that of dots, which implies that the friction for it must have been greater than I assumed; but for both the average curve is nearly the right line.

In applying minimum squares, my previous experience made me omit those where v was very small. I obtained—

First set	73.	$\alpha=12{\cdot}299$;	$\beta=17{\cdot}488$;	$\gamma=-15{\cdot}372$;	$\alpha'=11{\cdot}350$;	$\beta'=13{\cdot}401$.
Second set	39.	$\alpha=8{\cdot}721$;	$\beta=12{\cdot}974$;	$\gamma=-14{\cdot}397$;	$\alpha'=7{\cdot}753$;	$\beta'=8{\cdot}990$.
Total	112.	$\alpha=11{\cdot}183$;	$\beta=16{\cdot}954$;	$\gamma=-19{\cdot}862$;	$\alpha'=9{\cdot}880$;	$\beta'=11{\cdot}538$.

The second, as might be expected from the plotting, is not to be relied on; and in all of them the negative value of γ is too great. It will be seen in the subsequent values of $\Delta V'$ that those for May 26 are too small, so as to confirm my suspicion that the F which I used was too small. This may have been owing to the engine having been stopped during part of the time, or else doing lighter work. I have already mentioned that a set of measures taken by Mr. H. GRUBB while the engine was not acting gave larger results than those in Table IV. For B=36 the excess was 406·4; the $\Delta V'$ for 123 would correspond to an excess of 291. Under the circumstances this set by itself is useless; but as the earlier part of it seems correct, it may be retained in the total. But I give a double weight to the first. Using $\frac{9}{10}$ measured α as before.

First set 73	$\alpha=10{\cdot}706$;	$\beta=12{\cdot}328$;	$\gamma=0{\cdot}524$
,, reduced by (V) . .	,,	12·269 ;	0·937
,, omitting $v<3$ . .	,,	12·178 ;	0·936
Total 112	,,	13·193 ;	−2·991
	10·706	12·492	−0·148

Hence $x=1{\cdot}1648$; $y=-0{\cdot}0138$; $z=1{\cdot}3414$.

In both cases β seems tolerably certain; but it is not so for γ, especially in No. III.

The confusion of the summits of the curves (where the curvature is most influenced by γ) may account for this.

(40.) The near equality of β and γ for Nos. I. and III. seems to show that in these two anemometers they are nearly proportional to α. There should be a little excess in the first on account of the greater resistance of the arms, which are nearly twice as long, and instead of being (as they should be in working anemometers) of thin plates with sharp edges, are of $\frac{1}{2}$-inch tube. This must increase both β and γ in No. I. more than in No. III. Still the difference can be but very small, and I think that probably the mean values of them will satisfy both sets, within the limit of error of observation.

This may be tried by computing V′ by equation (III.) with the x and z deduced from their mean. We have on this assumption $x=1\cdot1282$; $z=1\cdot3400$.

ΔV′ the observed—the calculated V′ is given in Table XV.

TABLE XV.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
1	−0·18	34	+0·33	65	−1·26	97	+0·40
2	+0·93	35	−0·32	66	+0·23	98	+0·66
3	+0·77	36	−0·22	67	+0·65		
4	+1·28	37	−0·42	68	+0·11	99	+0·42
5	+1·45			69	+0·13	100	+1·47
6	+1·34	38	+0·06	70	+0·07	101	+1·55
7	+1·25	39	−0·03	71	+0·03	102	−0·45
8	+1·04	40	+0·30	72	−0·18	103	−0·10
9	+1·12	41	−0·25			104	+1·67
10	+0·25	42	−0·44	73	−1·79	105	+0·11
11	−0·23	43	+0·15	74	+1·44		
12	−0·27	44	−0·72	75	−1·17	106	+0·05
13	+0·55	45	−0·28	76	−0·34	107	+1·82
		46	−0·90	77	−1·08	108	+2·38
14	+0·08	47	−0·83	78	−1·25	109	+2·24
15	−0·38			79	−1·13	110	+1·82
16	−0·29	48	+0·25			111	+1·61
17	+0·11	49	+0·26	80	+0·43		
18	+0·04	50	+0·36	81	+1·01	112	+1·10
19	+0·37	51	+0·52	82	+1·10	113	+2·60
20	+0·47	52	+0·16	83	+1·07	114	+2·52
21	+0·23	53	+0·15	84	+0·99	115	+2·29
22	−0·29	54	+0·50	85	+0·67	116	+2·12
23	−0·34	55	−0·10	86	+0·17		
24	−0·61	56	+0·02	87	+0·26	117	+1·11
25	−0·94			88	−0·46	118	+3·45
26	−0·75	57	+1·05	89	+0·34	119	+2·79
		58	+0·94	90	+0·39	120	+2·77
27	−0·64	59	+0·24				
28	−0·65	60	+0·15	91	−0·22	121	+2·41
29	−0·49	61	+0·23	92	+0·80	122	+2·86
30	+0·01	62	−0·07	93	+1·11	123	+2·94
31	+1·30	63	−0·38	94	+1·19		
32	−0·12	64	+0·10	95	+1·20		
33	−0·02			96	+0·41		

The end of each series of a set is marked by a single line, and that of each of the two sets by a double one. It must be remembered that each set begins with a low value of v which increases in each term; in each also there is a different brake friction. The values of $\Delta V'$ are not very close; but this was to be expected from the uncertainty of W and F; and as the aberrant ones are not regularly distributed they are evidently due to casual errors. Thus in the first series of the first set, six have large positive values; but this is not the case in the corresponding terms of the other series, in which they are small and mostly negative, except the last, in which are four large negative and two positive.

In the second set the three first series are of the same character as the previous one; but there was evidently some disturbing influence at work in the last four. I have already mentioned what I believe this to have been.

The brake friction of the fourth occurs in the last but one of the first set, and the last of No. III., without producing such anomaly, so it is not due to error in the measure of the friction. The results for No. III. are given along with those obtained by supposing $\gamma=0$ in

TABLE XVI.

No.	$\Delta V'$.	Do. $\gamma=0$.
159	−0·43	−0·43
160	−0·17	−0·11
161	−0·27	−0·18
162	+0·08	+0·20
163	−0·08	+0·09
164	+0·33	..
165	−0·06	+0·28
166	−0·11	+0·14
167	+0·27	+0·58
168	−0·20	−0·29
169	+0·23	−0·25
170	−0·25	−0·30
171	−0·25	−0·22
172	−0·75	+0·20
173	−0·72	−0·56
174	−0·59	−0·03
175	−0·31	−0·30
176	−0·36	−0·33
177	+1·30	−0·99
178	−0·77	−0·66
179	−1·03	−0·63
180	−1·17	−0·98
181	−0·64	−0·68
182	+2·68	+2·68
183	+1·49	+1·20
184	−0·09	−0·00
185	−0·32	−0·20
186	−0·61	−0·45
187	−1·08	−0·69
188	+0·67	+0·90
189	−0·33	−0·07
190	+0·33	+0·63
191	−0·27	−0·02
192	−0·27	+0·06
193	+0·78	+0·70
194	−0·35	−0·25
195	−0·40	−0·31
196	+0·39	−0·36
197	−0·49	−0·45
198	−0·49	−0·41
199	−0·46	−0·35

Here also V′ is sufficiently well represented; but it is to be noted that $\gamma=0$ gives rather better results than the other. The probable error $=\pm0·384$, the maximum $=+2·68$, the minimum $=-0·98$. For the other the probable error $=\pm0·455$, the maximum also $=+2·68$, the minimum $=-1·17$. I did not think it necessary to compute the entire of No. I. on this hypothesis, but ten on which I tried it showed similar advantage.

(41.) I think these results warrant us to believe—first, that if the constants were properly determined, the equation (III.) with or without γ would give V′ with sufficient accuracy for all practical purposes; and secondly, that for 9-inch cups the same constants avail for arms of 24 and 12 inches. It also deserves notice that the substitution of $\frac{1}{2}$-inch tube for knife-edged arms in No. III., makes very little difference in the value of V′, as is seen by comparing 164 with 163.

(42.) By using the differential formulæ (V.) and (VI.) to correct the constants, the mean value of $\Delta V'$ might be considerably reduced; but I think this unnecessary, and I hope to get more accurate values of them by another process.

The plotting of No. II. is shown in Plate 67. It is far more confused than the previous two, and shows that the disturbing agents were still more powerful. It is not possible to detect the curvatures, and we can only infer that a right line can be drawn giving the average direction of the system, while the breadth of that system shows that we may expect the range of $\Delta V'$ to be considerable. This was to be expected, as the impelling force $\alpha\ (V'^2+v^2)$ is only one-fifth of its value in I., while the frictions are nearly the same, therefore any irregularity on the latter will be felt more powerfully; the same result will also be produced by the lesser momentum of No. II., which has less power to equalise fluctuation of motion. That such fluctuation exists is evident from examination of the chronograph sheets on which the time of each revolution of the anemometer is recorded. For instance, in 125 the maximum time of one revolution during the four minutes of the experiment was $4{\cdot}58^s$, giving $v=1{\cdot}87$; the minimum time $2{\cdot}39^s$, $v=3{\cdot}58$; and there were many intermediate values. For higher speeds the difference was not so great: in 132 the maximum time was $1{\cdot}06^s$, $v=8{\cdot}16$; minimum $0{\cdot}72^s$, $v=11{\cdot}97$. With 9-inch cups these variations are scarcely perceptible in the high speeds, but notable when v is very small. Thus in 182 maximum time is $6{\cdot}80^s$, $v=0{\cdot}62$; minimum $4{\cdot}50^s$, $v=0{\cdot}95$. In such cases it is evident that the mean v cannot give the mean V′, as m is not constant. If the arms of the anemometer had been of sharp plate, it is not improbable that the x and z of Nos. I. and III. might have availed here also; but as their section is 0·87 of the area of the 4-inch cups, γ, and in still greater degree β, bear a much greater proportion to α.

The character of the plotting prepared me for bad results from minimum squares: 34 gave $\alpha=3{\cdot}100$; $\beta=9{\cdot}6607$; $\gamma=-31{\cdot}140$, α cannot be larger than that given by my measures; and the γ would give V′ imaginary for v above 8. The result was not improved by omitting three of the smallest v's. Assuming $\gamma=0$, I had $\alpha'=1{\cdot}681$; $\beta'=2{\cdot}602$; both evidently too small. I then tried, as in previous cases, nine-tenths of my measured α, and had by six various combinations, $\alpha=2{\cdot}084$; $\beta=4{\cdot}4065$; $\gamma=-7{\cdot}626$. Still, as formerly remarked, γ is too large, and avoiding it entirely I got $\alpha'=2{\cdot}084$; $\beta'=3{\cdot}489$. Then taking twelve distributed over the entire set, I computed the values of $\Delta V'$ for them, and from these found by (VI.) $y=-0{\cdot}8146$; $x=1{\cdot}7390$; $z=2{\cdot}209$. The results are given in

TABLE XVII.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
124	+0·54	134	+0·75	144	−0·04	152	+0·63
125	+1·24	135	+0·37	145	−0·82	153	−0·66
126	+1·40	136	+1·17	146	−1·09	154	−0·62
127	+1·94	137	+1·00			155	−1·19
128	+1·35	138	+1·26				
129	+2·18	139	+1·33	147	−0·54		
130	+1·95	140	−0·22	148	+0·87	156	+0·38
131	+1·81			149	−0·15	157	−1·40
132	+0·63	141	+0·33	150	−0·59	158	−1·51
133	−0·27	142	+1·10	151	−1·43		
		143	+0·04				

Here also the entire set is nearly as well represented with $\gamma=0$ as by the entire equation. The probable error of Table XVI. $=\pm 0{\cdot}630$. The maximum value of $\Delta V'$ in it $=+2{\cdot}18$; the minimum $=-1{\cdot}51$. In the other case the probable error $=\pm 0{\cdot}637$; the maximum $=+1{\cdot}66$; the minimum $=-2{\cdot}18124$. It was observed on a different day from the rest of that series, and its difference from its neighbours seems to imply an excess of friction in the latter. The table might evidently be improved by farther approximation.

(43.) The plotting of No. IV., Plate 69, resembles the other except in one notable circumstance. The dots of the first series (200 to 210) are lower than the rest, although their general direction is nearly parallel to that of the others. This indicates that the friction during their course was greater than on the following days. This may have been caused by bad oil, as already mentioned, and a rough measure of f taken then gave it $=250$ grains instead of 185, which I computed it to be from the disc measures. The viscidity of the oil would increase f; but if, asis probable, it was also applied to the pivots of the brake, it would lessen the pressure of the rubbers and the brake friction. Both these effects seem to have taken place to such an extent that I find it impossible to derive from the observations any constants which will represent the entire set. It was scarcely to be hoped that minimum squares could give a good result. I tried them, omitting the first 11, and had $\alpha=3{\cdot}682$; $\beta=7{\cdot}541$; $\gamma=-18{\cdot}172$. These are inadmissible; α is too far above my measures, and γ far too negative. Assuming $\gamma=0$ the result is no better. Taking, as in the others, $\alpha=0{\cdot}9$, the measured one for II., the result is still worse. It seemed however possible that though α and β were both astray, their ratio might approximate to the truth. This gives $x=1{\cdot}0240$; $x^2=z=1{\cdot}049$. I computed by these $\Delta V'$, intending to correct them by the formula (VI.). The second series was well represented; in the first the V' was too small; in the rest too great, and it was not possible by any correction of the constants to represent the whole. The constants just given for No. II. failed utterly; but to my surprise those used for Nos. I. and III. were not inferior to the preceding. I have little doubt that with the true frictions they would give correct results. Aberrant as both are, I give the results in—

Table XVIII.

No.	ΔV′	Do. I. and III.	No.	ΔV′.	Do. I. and III.
200	+0·22	−0·02	217	+0·92	+0·47
201	+2·46	+1·15	218	+0·95	+0·55
202	+2·38	+1·72			
203	+3·24	+2·46	219	−2·83	−2·99
204	+3·20	+2·28	220	−2·21	−2·66
205	+3·29	+2·29	221	−1·96	−2·58
206	+3·56	+2·49	222	−1·37	−2·00
207	+4·01	+2·76	223	−1·38	−2·35
208	+4·16	+2·67	224	−0·44	−1·64
209	+4·40	+3·04			
210	+4·50	+2·98	225	−2·57	−3·08
			226	−2·47	−2·18
211	+0·47	−0·23	227	−1·01	−2·80
212	−0·70	−1·39	228	−2·08	−2·95
213	+0·09	−0·80			
214	+0·12	−0·79	229	−3·02	−3·17
215	+0·70	−0·51	230	−2·81	−2·96
216	+1·00	−0·27	231	−2·42	−3·01

If, as I suppose, the viscid oil increased f and f''' and lessened the brake frictions, there must be a certain part of the set where these opposite effects will balance each other; and this seems to occur in the second series. The increase of F which would nullify ΔV′ for 210 in the second system =139; that for 231=−225; both quite possible.

(44.) No. V. cylinder cups is in marked contrast to No. IV. Its plotting (Plate 70) is more regular than any of the others. Minimum squares give $\alpha=9{\cdot}152$; $\beta=13{\cdot}416$; $\gamma=-3{\cdot}481$. The α is a little less than what I got by two doubtful measures, so I retain it, and have $x=1{\cdot}4659$; $z=1{\cdot}769$. The results of these are given in—

Table XIX.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
232	−0·81	240	−0·24	247	−0·49	254	−0·57
233	+0·39	241	−0·27	248	−0·56		
234	−0·19	242	−0·60	249	−0·51		
235	−0·03	243	−0·42	250	−0·33	255	−0·63
236	+0·12	244	−0·62			256	−1·06
237	+0·26	245	−0·59			257	−1·03
238	+0·18			251	+0·70		
239	+0·14			252	−0·36		
		246	−1·57	253	−1·16		

These might be considerably improved by a small increase of α; for instance, if I had taken it $=0{\cdot}9\times10{\cdot}4$, the error for 257 would have become $-0{\cdot}66$.

(45.) I was surprised at finding the cylinders so much more consistent than the hemispheres. I attribute it to two causes: 1, the cylinders are 18 oz. heavier than the 9-inch hemispheres, and their greater momentum tends to neutralise any fluctuations of friction or of the vortex's current; 2, any radial currents will probably have more power to increase the resistance by acting on the convex hemispheres than they can have on the planes which terminate the cylinders. It must be observed that in this form of anemometer the lateral friction is greater than with the hemispheres.

$D=75$ gave $V'=22\cdot5$, while with No. 1 it was $27\cdot6$.

I shall now briefly recapitulate the conclusions which, in my opinion, may be drawn from these experiments.

(46.) The equation (III.) represents the observations from $V'=5$ to $V'=40$ and through a wide range of friction. The discordances follow no law which might indicate a relation between V' and v different from that expressed by (III.); they are evidently such as would be produced by casual disturbances of W and F.

(47.) If the equation (I.) contains any other functions of V' and v than squares and products, their effect is so small that it is masked by these disturbances.

(48.) The coefficient of the impelling forces α appears to be as the area of the cups, and to be the same for radii of 24 and 12 inches.

(49.) There is reason to believe that β and γ are proportional to α, or that x and z are constant in all anemometers whose arms are of sufficient length to prevent the wake of one cup from interfering with the cup which follows it. For Nos. I. and III. this is highly probable; it is not disproved by No. IV., which is not worse represented by it than by any other system of constants. No. II. requires a larger x and z than the others, but I think I have assigned a sufficient cause for this. If this opinion be well founded, the difference between the indications of different anemometers would depend only on the fraction $\frac{F}{\alpha}$, and the limiting value of $\frac{V'}{v}$ given by the constants of paragraph 39 would $=2\cdot286$ instead of 3, which I originally assigned to it. More exact observations may show that x and z are not rigorously constant, and diminished section of the arms may alter this limit; but probably the changes will not be of much practical importance.

(50.) If these observations were repeated in a quiescent locality, if the frictions were measured at each experiment and by the means described in the note to paragraph 20, more accurate results would certainly be obtained; but there would remain the inevitable difficulty arising from the impossibility of rightly estimating the effect of W. Any measures of it which can be taken give only an average of the whole circumference, and one outside the anemometer's track, while the experiments with the balloons and air-meter show that it varies notably in various parts of the circumference, and with the distance from the centre. There is also an outward current of considerable magnitude, but very irregular. The rotation of the anemometer itself, whether vertical or horizontal, produces a secondary vortex, which must modify the primary one; to what degree we cannot say. The effect of these variations of W is shown by the changes of v in individual revolutions, while V is perfectly uniform.

(51.) For this reason, combined with the difficulty of obtaining satisfactory measures of the centrifugal friction, I have given up the idea of repeating these observations, and I propose instead to try the method described in the end of my paper.

I have established on the flat roof of the dwelling house here the anemometer No. I., with its axis vertical and 22 feet due east of that of the Kew instrument already there. Both of them are about 16 feet above the roof. The chronograph and its battery of six LECLANCHÉ'S cells are placed in the wooden house which supports it, and are connected with it and the other instrument by insulated wire and contact makers. From the altered position of the axis it is necessary to apply the brake weights by means of a right-angled lever; the counterpoises already described have been replaced by springs to prevent the chance of accidental disturbance of their position. The axle now rests on a toe of hard steel, 0·25 inch in diameter, turning in oil; and to prevent it from cutting under the pressure of the axle and its appendages, the rollers formerly used to lighten the centrifugal friction have been altered by Mr. H. GRUBB so as to relieve the toe of any required portion of the weight. This arrangement works so well that the friction is little more than in the Kew one, though its weight is 1·6 times as great. The arms are now of plate steel $\frac{1}{8}$th of an inch thick, with a sharp edge, strengthened by wire stays, and with means of placing both the 9-inch and 4-inch cups at 24, 12, and 8 inches from the centre.

(52.) The mode of experimenting which I intend to follow is this. Considering the Kew as the standard S, and the other E, and arranging one of the chronograph points to register the v of S, the other of E, apply a brake friction to the latter which will diminish v. When this has lasted long enough to make it tolerably certain that each instrument has been acted on by the same amount of wind, apply a different brake friction to E and take another set. We have hence three equations, but four unknown quantities, V, α, β, and γ; and since α appears as a coefficient of V^2, it cannot be obtained separately. But if, as I believe, it is given with tolerable precision by the measures given in Table VI., the others may be determined.

The formula (VI.) enables us to estimate for the Kew anemometer the error in V caused by $\Delta\alpha=1$. When $v=1$ it is $-0·20$, 0·059 of the computed V; for $v=20$ it is $-3·30$, 0·072 of V. The highest v I ever observed was in a squall in March, 1849, when during five minutes it was 41·2; this would give $\Delta V=-8·14$, 0·072 of 112·80. It is possible that more exact measures of α might be obtained by moving the measuring apparatus in a right line, and in an enclosed space of sufficient dimensions, but the requisite conditions for such an experiment will not be easily realised.

(53.) If these constants give for any other pair taken with a different wind $V_s=V_E$ they are right; if not, the formula (VI.) will show us whether the error arises from using a wrong α, or whether the constants vary with v.

(54.) It is not easy to tell beforehand what difficulties may beset this mode of investigation. The most obvious one is the irregularity of wind which may be expected to vary from one anemometer to the other, and also at each during an experiment. If it blow in the direction W.E., or *vice versâ*, there is danger that the eddy caused by the windward one may reach the others; if it be S.N., different streams of the current may fall on each. The extent of these disturbances may be studied by making the friction of E equal to that of S; and it will also show what length of time is required to make the average V the same in both. The changes of V during the experiment may, I think, be eliminated by sorting the two v into sets, of which the individuals are all in the same proportion, and comparing *them* separately. This can easily be done by measuring the intervals on the chronograph sheets.

How far these precautions will avail can only be ascertained by trial, but I hope that it may be given to me to make this trial to its full and decisive extent.

APPENDIX.

The object of the experiments being to determine the relation between the velocity of actual wind supposed uniform (the air also being at, or reduced to, a normal density), the velocity of the cups and the friction, I assume in the first instance as correct, the values of those two quantites given by the experiments with the whirling machine, and proceed to consider the relation.

Let V' be the velocity with which the air passes the anemometer, that is, in the case of the actual experiments, the velocity of the centre of the anemometer itself corrected for the velocity of the wind produced by it; let v be the velocity of the centre of the cups, F the moment of the total friction. Then supposing the density of the air normal for a given anemometer, v will depend only on V' and F, that is, there will be a functional relation between the three variables V', v, F, leaving two of them independent.

In investigating experimentally the relation between two variables, it is often very useful to plot the results of experiment, as the general character of the relation sought, and the allowance to be made for errors of observation can thus be estimated. The relation between three variables would be expressed graphically by a surface instead of a curve, and it is troublesome to model a surface. If, however, we can find a relation between the variables which is satisfied, *provided* some other relation is satisfied, we can thereby reduce the number of independent variables from two to one, and employ ordinary plotting in investigating the relation between the variables. In fact, the relation sought is reduced from one of the form $V'=\phi(v, F)$ to one of the form $f(V', v, F)=\psi\{f_1(V', v, F)\}$, where ϕ, ψ denote unknown, and f, f_1 known functions.

In the present case, since by hypothesis the anemometer is in a permanent state, the moment of the friction is equal to the *total* impelling force of the air, *i.e.*, the total pressure arising from the motion of the air, without distinction of impelling or retarding, but reckoning the latter as a negative impelling force. Now, in cases of rapid motion like that of the air passing the cups of the anemometer, it is well ascertained that the resistance varies as the square of the velocity, all other circumstances being the same. Hence, with a given anemometer, when only the scale of the velocities changes, *i.e.*, when V varies as v, the moment of the total impelling force may be expected to vary as the square of the velocity. When the density changes it may be expected also to vary as the density. Hence we may expect that when v varies as V′, then F varies as $\rho V'^2$, or in other words that

$$F=\rho V'^2\phi\left(\frac{v}{V'}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϕ denotes some function the form of which is not at present under consideration.

Let $\frac{v}{V'}=\xi$, $\frac{F}{\rho V'^2}=\eta$, and for each observation let the point whose coordinates are ξ, η be laid down on paper. If $F\rho^{-1}$ were merely some arbitrary function of V′ and v, the points so laid down would be spread out over the paper, but if equation (1) be true they will lie in a definite curve.

The actual experiments were executed in series, in each of which only one independent variable was changed, so that if the experiments were infinitely numerous and infinitely exact the locus of the point whose coordinates are ξ, η would be a definite continuous curve. And the test of the truth of (1) is that the curves belonging to the different series shall coincide, instead of being arranged in some order of sequence.

Plate 68 shows the result of plotting the observations taken with anemometer No. III. On inspecting the figure it will be seen that the different series fit very well into one another. Departures there are no doubt in the individual observations from a mean curve, but these appear to be casual, not methodical and depending upon the order of the series.

The result of the observations then is confirmatory of the fundamental supposition made hitherto, that when the friction is so arranged that the velocity of the air passing the anemometer bears a given ratio to the velocity of the cups, the moment of the total impelling force varies as the square of either velocity.

Assuming then the truth of equation (1),* we have next to inquire what is the form of the function ϕ?

* It formed no part of the object of the experiments to determine the relation of the impelling force to ρ, which merely comes in as a small correction for reducing observations made on different days to a common standard. It is the dependence of F on V′ and v that is contemplated in the text.

A complete hydrodynamical solution of the problem is altogether beyond our power. On the other hand, the irregularities of the observations prevent us from going, by observation alone, more than a certain way towards the determination. We must, therefore, endeavour to combine as best may be the indications of mechanical theory with the results of experiments.

In his paper "On the Cup Anemometer," in the Transactions of the Royal Irish Academy, Dr. ROBINSON has shown that (supposing the density constant, say $=1$) the relation between the moment of the impelling force and the moment of the friction is either accurately or approximately of the form

$$F=\alpha V'^2-2\beta V'v-\gamma v^2, \quad \dots \dots \dots \quad (2)$$

which would give for the locus of the point whose coordinates are ξ, η the parobola

$$\eta=\alpha-2\beta\xi-\gamma\xi^2. \quad \dots \dots \dots \quad (3)$$

If now we turn to the plotting of the observations, we see that the best smooth curve to represent the observations, free from sinuosities which the observations do not warrant us in supposing real, is either accurately or approximately a straight line,

$$\eta=\alpha'-2\beta'\xi; \quad \dots \dots \dots \quad (4)$$

in fact, so nearly does a straight line represent the observations that it is not easy to say to which side the concavity of the line, if curved it be, should lie. On the whole there appears to be a slight indication of a gentle concavity *towards* the origin.

It may be remarked in passing that the formula (4) which the experiments show to be at least very approximately true, leads to a very simple expression for v in terms of V', namely—

$$v=aV'-\frac{b}{V'},$$

where a and b are constants.

The figure shows that there cannot be much doubt as to the distance from the origin at which the curve intersects the axis of ξ, nor as to the direction of the curve at that point; and generally that the curve is well determined in its right hand half, though it becomes more uncertain towards the left. If λ be the value of ξ at the point of intersection, and $-t$ the tangent of the inclination at that point, the equation of the curve, assumed to be a parabola, will be

$$\eta=t(\lambda-\xi)-C(\lambda-\xi)^2; \quad \dots \dots \dots \quad (5)$$

or again, if we suppose known two points (p, h) and (q, k) lying in the well-determined part of the curve, its equation will be

$$\eta=(q-p)^{-1}\{h(q-\xi)+k(\xi-p)-C'(\xi-p)(\xi-q)\}; \quad \dots \dots \quad (6)$$

and as $(\lambda-\xi)^2$ or $(\xi-p)$ $(\xi-q)$ will be small throughout the well-determined part of the curve, the constant C or C′ will admit of considerable latitude of variation without much affecting the satisfaction of the observations. Conversely, if we attempt to determine the constant C or C′ from the observations, in addition to the two elements λ, t, or h, k, the determination will be extremely precarious. And if we arrange the formula (5) or (6) according to powers of ξ, so as to throw it into the form (3), the precariousness of the determination of C or C′ will more or less affect all the three constants α, β, γ.

Accordingly, if we take this formula, and attempt to determine the three constants α, β, γ, from the observations, it may be that by different processes we shall arrive at results differing considerably, not only as regards γ, but even, though to a less degree, as regards α and β. It is not until we use the values of α, β, γ, so obtained for the determination of two out of the three elements of the parabola which are well or fairly determined by the observations, that we perceive the accordance which underlies the apparent discrepancy.

If the simple formula (4) so nearly fits the observations, it is by no means *merely* as an empirical formula of interpolation presenting two arbitrary constants whereby an approximate accordance may be brought about, or in the way that a small arc of an arbitrary curve may be approximately represented by a straight line. The observations were also plotted by taking for coordinates $V':v$ and $F:v^2$ instead of $v:V'$ and $F:V'^2$, and in this case the curvature of the curve was very decided. Accordingly, though the observations may be satisfied by the first two terms of the formula (2) almost as well as by the three, that is by no means true of the last two, though in both cases alike we have two arbitrary constants at our disposal.

Explanation of the Plates.

In these plates the results of the experiments made with the five anemometers are graphically represented. In the plottings the abscissa is throughout $20v \div V'$. In Nos. I. and III. the ordinate represents $(f+B'+f''') \div \rho V'^2$; in Nos. II., IV., and V. it represents the same divided by the ratio of the area of the mouth of the cups to that of the mouth of the cups in Nos. I. or III.; that is, it represents the above expression multiplied by $81 \div 16$ for Nos. II. and IV., or by $\pi \div 4$ for No. V.

In the plottings, 0·75 inch is taken as the unit.

The reference numbers represent the order of the series, 1 meaning no weight on the brake, 2 the lowest weight, and so on.

In the experiments with anemometer No. I., the full dots represent the first set, experiments Nos. 1 to 79; No. 45 being omitted for a reason already stated. The small circles refer to the second set, experiments Nos. 80 to 123.

In all the plottings, except that for No. IV., a parabola or straight line, or both a parabola and a straight line, are laid down for comparison. The elements chosen for these are as follows :—

For No. I., $\alpha = 11 \cdot 183$, $\beta = 16 \cdot 954$, $\gamma = -19 \cdot 862$, $\alpha' = 9 \cdot 880$, $\beta' = 11 \cdot 538$.

For No. II., $\alpha' = 2 \cdot 084$, $\beta' = 3 \cdot 489$.

For No. III., $\alpha = 9 \cdot 472$, $\beta = 8 \cdot 469$, $\gamma = 9 \cdot 814$, $\alpha' = 9 \cdot 989$, $\beta' = 10 \cdot 928$.

For No. V., $\alpha = 9 \cdot 152$, $\beta = 13 \cdot 416$, $\gamma = -3 \cdot 481$.

The ordinates of the straight line and parabola were of course reduced in the same ratios as those of the dots, giving for the reduced constants in their equations—

For No. II., $\alpha' = 10 \cdot 550$, $\beta' = 17 \cdot 663$.

For No. V., $\alpha = 7 \cdot 188$, $\beta = 10 \cdot 537$, $\gamma = -2 \cdot 734$.

PLATE 67.

Fig. 1. Section of the horizontal arm (paragraph 2).

Fig. 2. The driving apparatus (paragraph 6).

PLATE 70.

Fig. 3. Apparatus for measuring the vortex current (paragraph 13).

Fig. 4. Brake apparatus (paragraph 21).

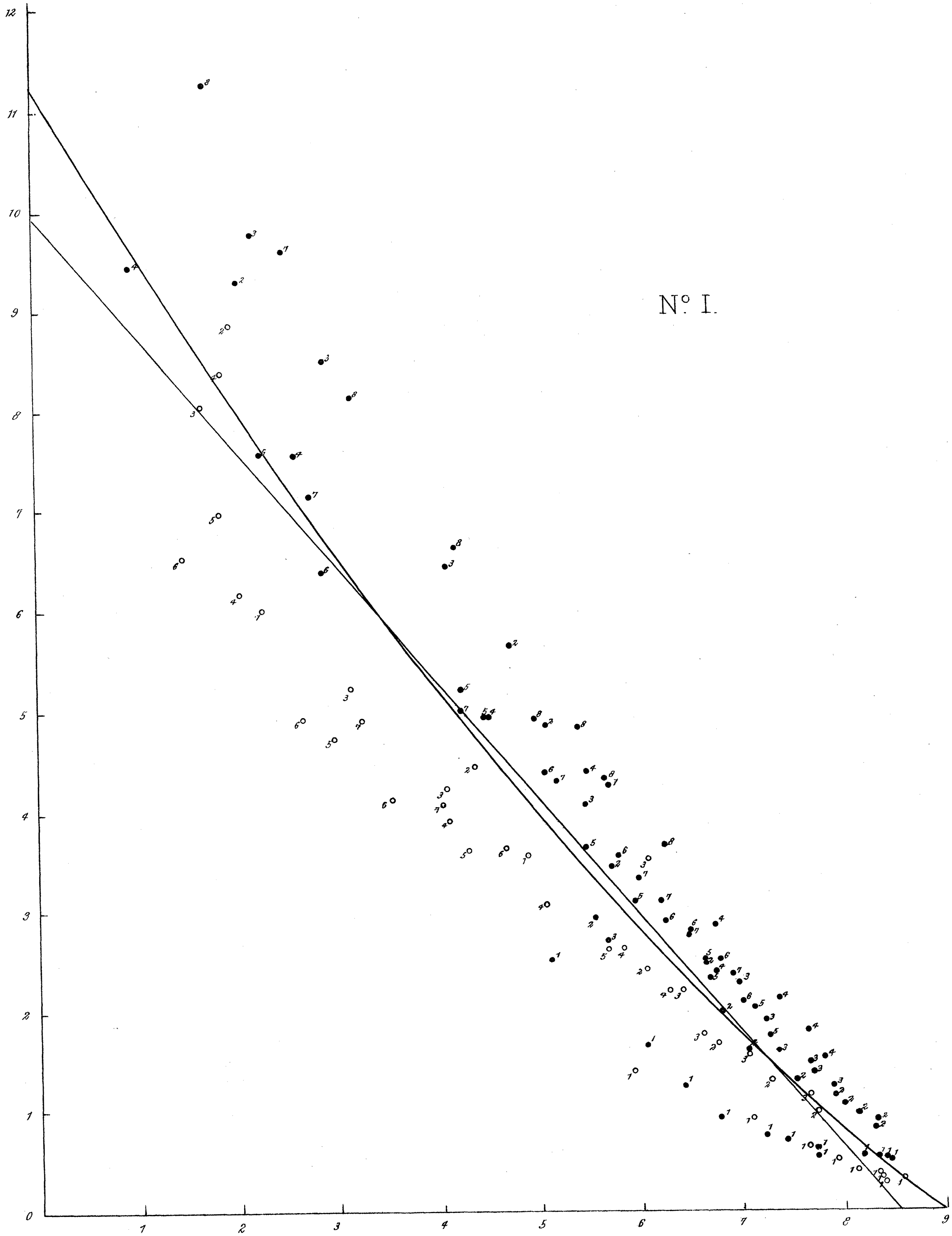


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Fig. 1.

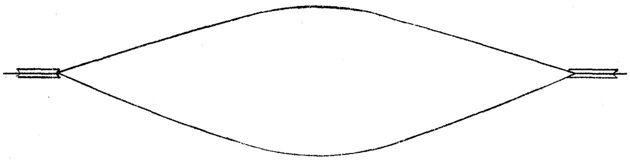
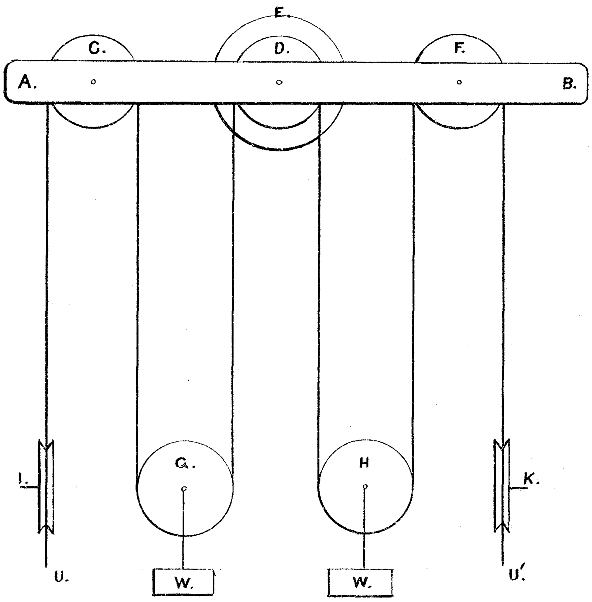
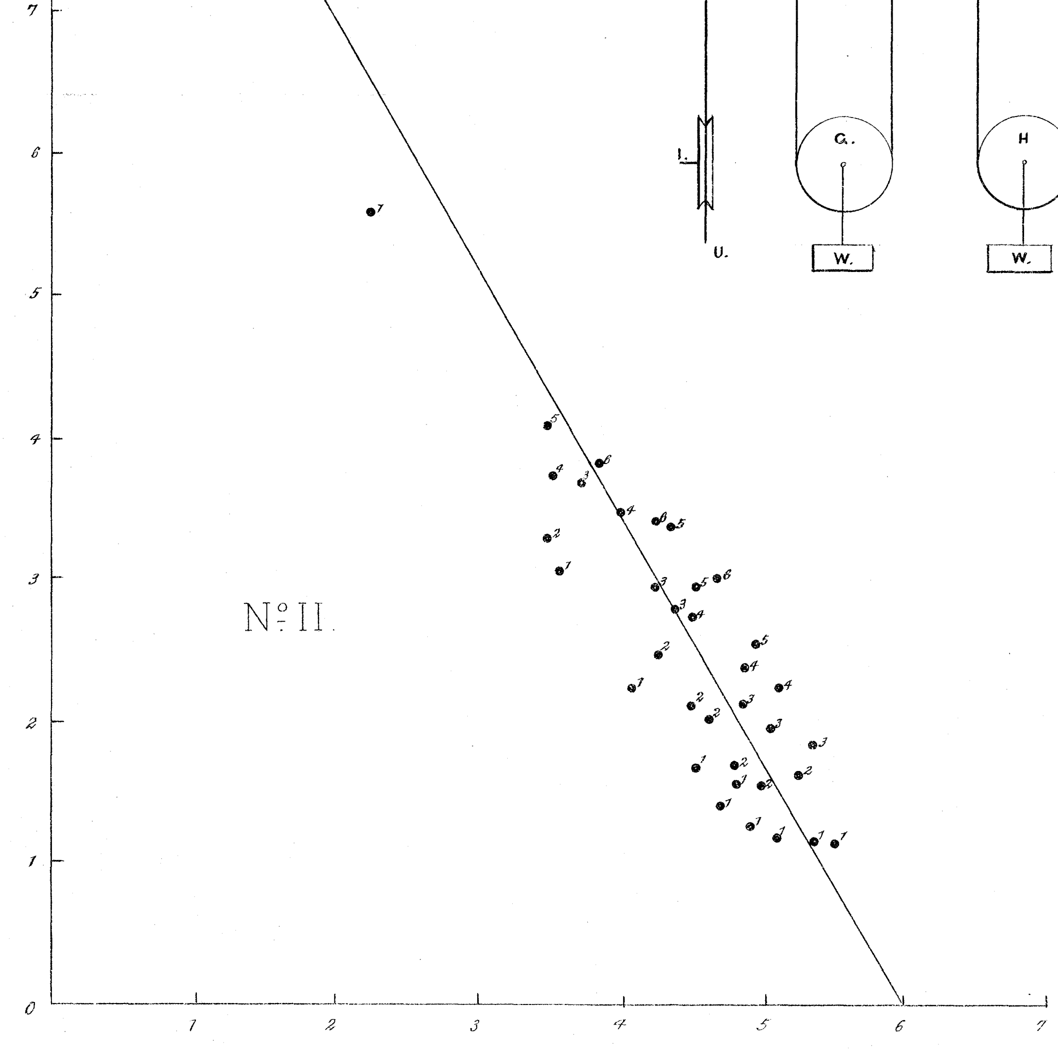


Fig. 2.





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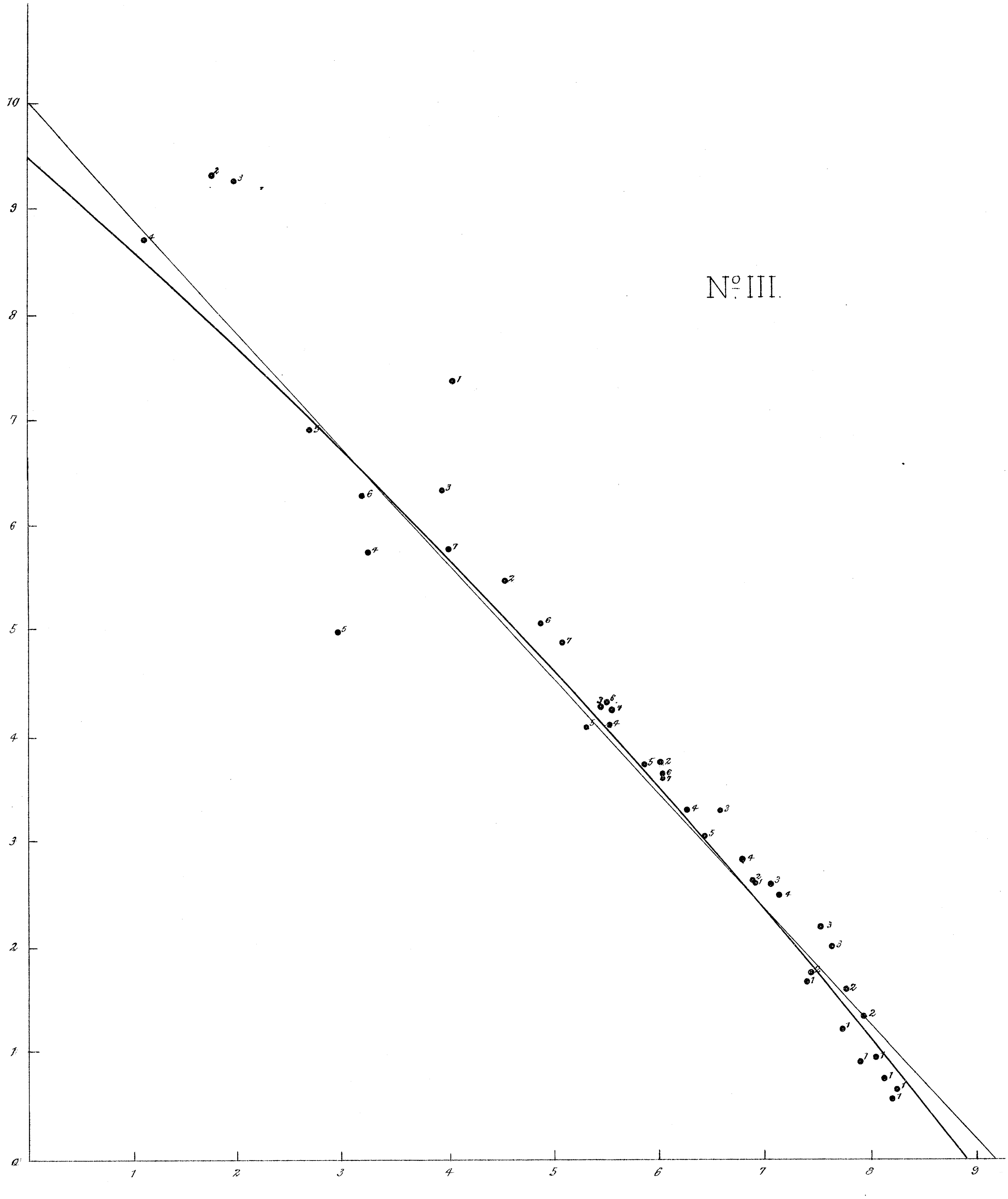

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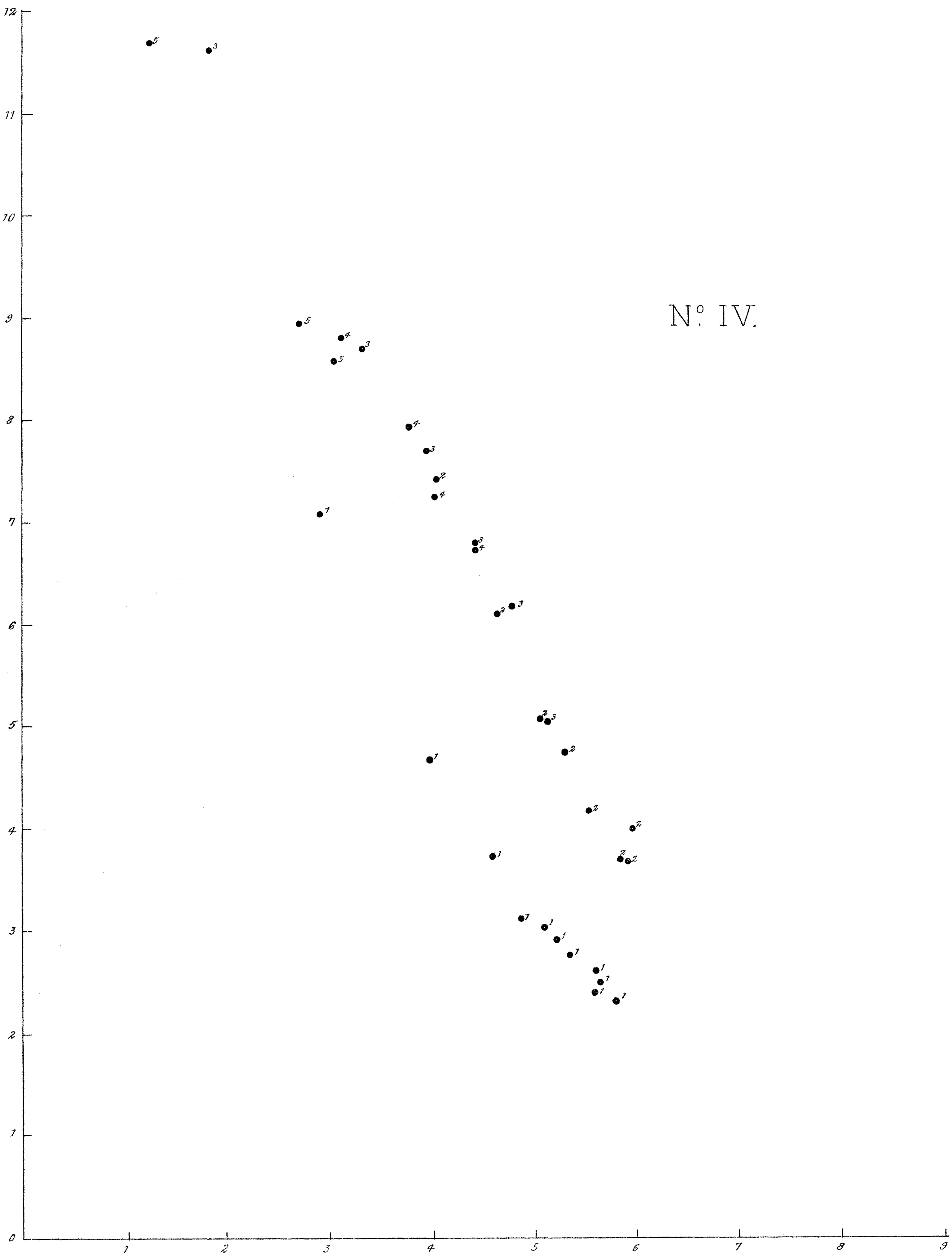

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Fig. 3.

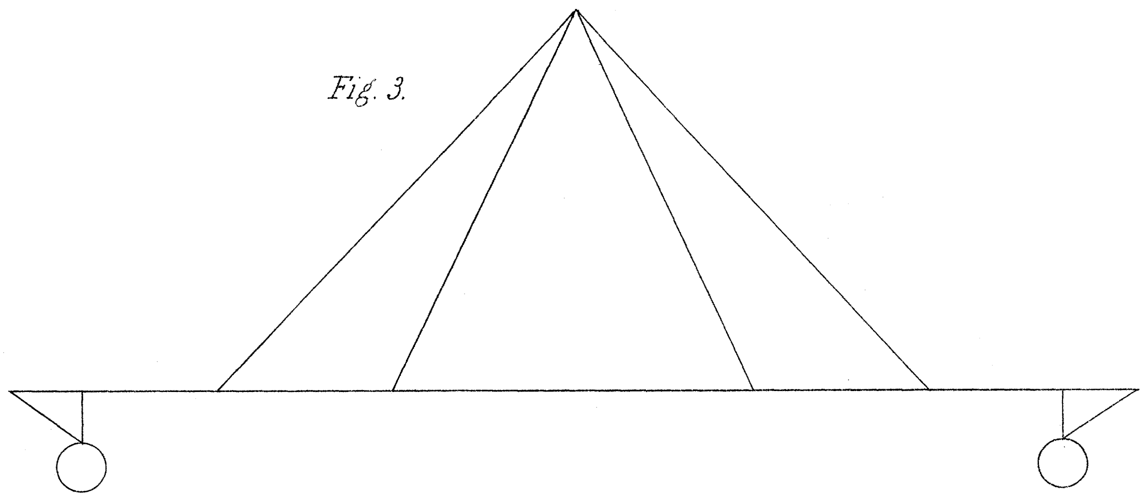
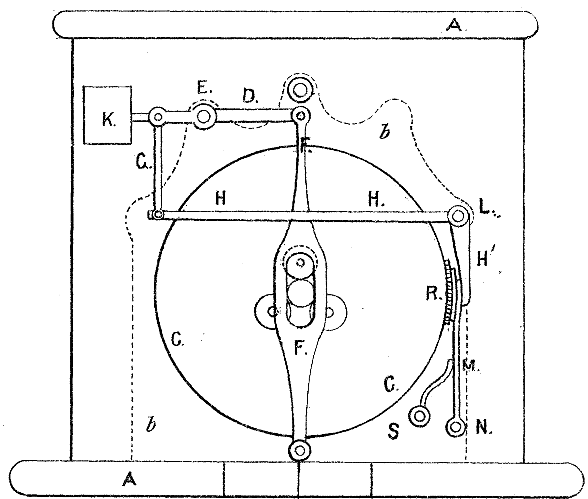
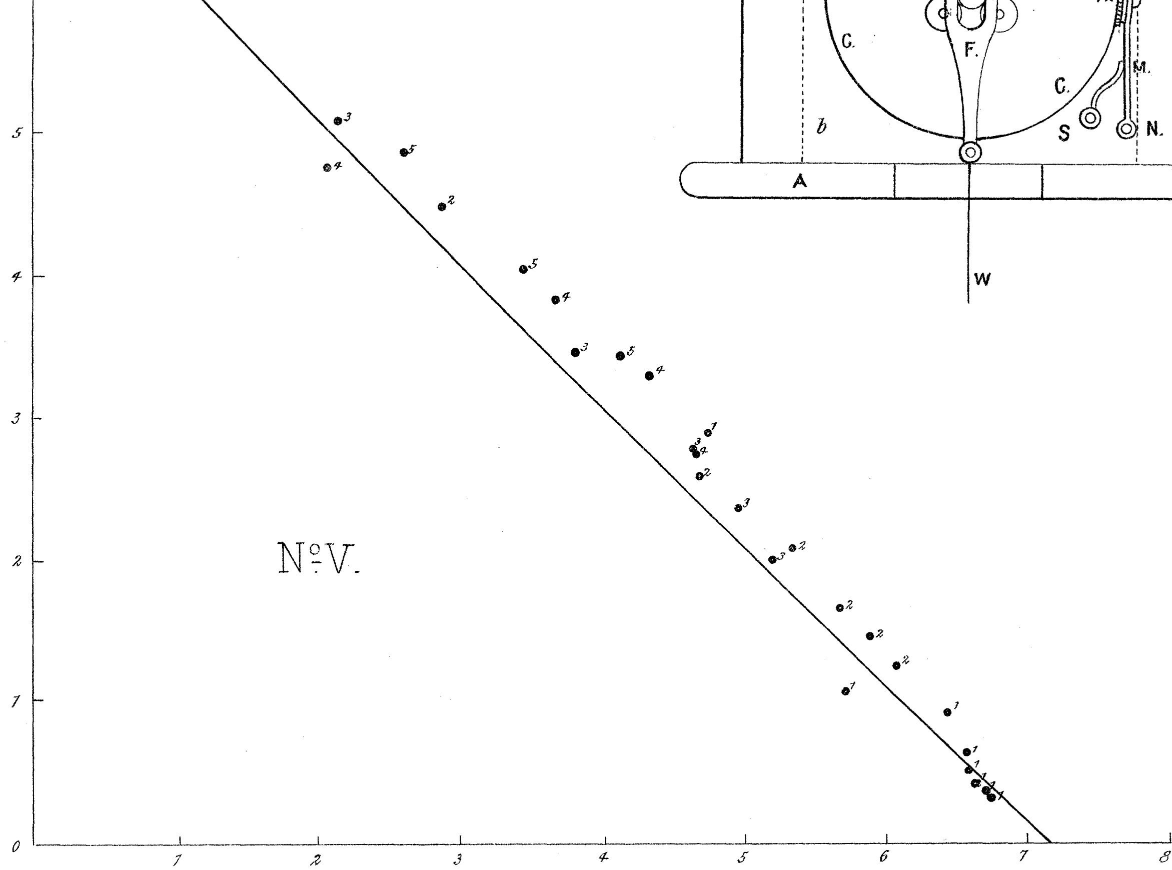


Fig. 4.





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